Spin field for N=1 particles

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Intro

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- Twisting/deforming the BRST differential by background fields $\Rightarrow$ dynamical e.o.m. (on a suitable rep. space)

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- **RNS particle**:
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  - $N = 4$: Einstein's gravity [Bonezzi, Meyer, Sachs, 2018] & NS-NS sector of Sugra (metric, B-field, dilaton) [Bonezzi, Meyer, Sachs, 2020]

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What about R-R fields? $\Rightarrow$ *need for spin field*

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Super-worldsheet

- Super-Virasoro algebra $\Longrightarrow$ the fermionic components of the superfield are 2d spinors and thus double-valued

$$\varphi^{NS}(e^{2\mathrm{i}\pi}z) = +\varphi^{NS}(z)$$
$$\varphi^{R}(e^{2\mathrm{i}\pi}z) = -\varphi^{R}(z)$$

- R states are created from the NS vacuum by a spin field $\vartheta(z)$

- $\vartheta(z)$ may be represented as endpoint of a branch cut in the Grassmann odd fields

[Friedan, Martinec, Shenker, 1986]

Super-worldline

A particle can be seen as the zero mode of a string

*Could there be a **spin field** operator for the particle?*

Problem:

- no branch cuts for $\mathbb{R}^{1|1}$

(we set d=4 in the target)

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Idea (inspired by [Sorokin, Tkach, Volkov, Zheltukhin, 1989]):
use a Grassmann degree shifting involution $\uparrow$ and six Weyl spinors $\vartheta_\alpha, \tilde{\vartheta}^{\dot\alpha}, \varepsilon_\alpha, \tilde{\varepsilon}^{\dot\alpha}, \lambda^\alpha, \tilde{\lambda}_{\dot\alpha}$
with a product $\circ$ inducing the *sole* commutators:

$$[\vartheta_\alpha, \lambda^\beta] = \delta_\alpha^\beta = [\varepsilon_\alpha, \lambda^\beta], \qquad [\lambda^\alpha, \uparrow] = 0 = [\lambda^\alpha, \lambda^\beta],$$

(+ antichiral counterparts)

Clifford algebra

With the previous relations, the algebra for a particle on a superworldline in 4d

$$[x^\mu, p_\nu] = \delta^\mu_\nu, \quad \{\psi^\mu, \psi^\nu\} = 2\eta^{\mu\nu}$$

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remains defined if the Gamma matrices are represented as:

$$\psi^\mu := \vartheta^\alpha \sigma^\mu_{\alpha\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}} \uparrow + \tilde{\vartheta}_{\dot{\alpha}} \tilde{\sigma}^{\mu\,\dot{\alpha}\beta} \lambda_\beta \uparrow \tag{1}$$

- The right (anti)commutators relation are satisfied up to an operator $\mathcal{I} := \vartheta \cdot \lambda + \tilde{\vartheta} \cdot \tilde{\lambda}$, and a correction $f^{(\mu\nu)}(\vartheta, \tilde{\vartheta}, \uparrow, \lambda, \tilde{\lambda})$

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- The right (anti)commutators relation are satisfied up to an operator $\mathcal{I} := \vartheta \cdot \lambda + \tilde{\vartheta} \cdot \tilde{\lambda}$, and a correction $f^{(\mu\nu)}(\vartheta, \tilde{\vartheta}, \uparrow, \lambda, \tilde{\lambda})$
- The operator $f^{\mu\nu}$ vanishes if the iterated *adjoint* action of λ a/o $\tilde{\lambda}$ on every state in the representation space is zero

States

- Supposing that there is a (NS) vacuum, the Weyl spinors ϑ, ε, $\tilde{\vartheta}$ and $\tilde{\varepsilon}$ are spin fields
- A Hilbert space annihilated by iterated action of λ a/o $\tilde{\lambda}$, or equivalently that survives projection into $\ker \mathcal{I} - 1$, consists of:
  - $|\varphi\rangle = \varphi_\alpha(x)\vartheta^\alpha \begin{pmatrix} |0\rangle \\ \uparrow |0\rangle \end{pmatrix}$, + anti-chiral state (1-particle)

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  - $C_{\alpha\beta}(x) \underbrace{\vartheta^\alpha \uparrow \left(\vartheta^\beta - \varepsilon^\beta\right) \begin{pmatrix} |0\rangle \\ \uparrow |0\rangle \end{pmatrix}}_{=:|e^{\alpha\beta}\rangle}$, $\tilde{C}^{\dot\alpha}{}_\beta(x) \underbrace{\tilde{\vartheta}_{\dot\alpha} \uparrow \left(\vartheta^\beta - \varepsilon^\beta\right) \begin{pmatrix} |0\rangle \\ \uparrow |0\rangle \end{pmatrix}}_{=:|e_{\dot\alpha}{}^\beta\rangle}$,

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    + other combinations of different chiralities (2-particles)
  - 3-particles, 4-particles, ...

We got spinors, bi-spinors etc. in the space of states

Do they represent RR-fields?

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- Worldline/worldsheet description is useful for two main reasons:
  1. study of backgrounds by deformation/twist of BRST differential;
  2. calculation of tree-level and 1-loop amplitudes for given vertices.

Super-worldline BRST

- A worldline with super-reparametrization invariance:

$$\{q, q\} = H, \quad q := \psi^\mu p_\mu, \quad H := p^2 \,.$$

Corresponding BRST operator $Q = cH + \gamma q - \gamma^2 b$ with ghost-antighost pairs (c, b), (γ, β) and algebra:

$$[\gamma, \beta] = 1 = \{c, b\} \,.$$

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- ψ^μ taken to be (1) and the space of states as previously constructed $\Leftrightarrow$ Hamiltonian constraints/momentum mapping in BRST operator.

BRST Cohomology

Choose to represent this BRST algebra on wavefunctions $\Psi(x, \vartheta(\tilde{\vartheta}), \varepsilon(\tilde{\varepsilon}), \gamma, c)$

- in the 1-particles sector, $|\Psi\rangle := \sum_k \gamma^k \left(|\varphi\rangle_k + c\,|\varphi^c\rangle_k\right)$: $Q\,|\Psi\rangle = 0 \;\Leftrightarrow$ one independent equation, *Weyl (anti-Weyl) equation*:

$$\not{\partial}\,|\varphi\rangle_0 = 0.$$

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- in the 2-particles sector, which comprise 0-forms, 1-forms and 2-forms, independent equations at ghost number 0 from Fierz identities are:

$$\mathrm{d}F^{(1)} = 0 = \delta F^{(1)}$$

$$\mathrm{d}F^{(0)} + \mathrm{i}\delta F^{(2)} - \frac{1}{2} \star \mathrm{d}F^{(2)} = 0$$

For $F^{(0)} = 0$, these are compatible with **linearised R-R field equations**.
Notice that $\mathrm{Im}Q \ni |\zeta\rangle_{k\geq 1}, |\zeta^c\rangle_{k\geq 1}$ *(ghost zero states are never Q-exact)*.

To analyse backgrounds via deformations of the BRST differential

$$\Downarrow$$

Better to focus on the chiral supercharge!

Chiral sector and an equivalent $H^{\bullet}$

Our observations concerned the chiral and antichiral sector on the same footing. Abandon reality of ψ^{μ} and focus on chiral supercharge:

$$\mathbf{q} := \tilde{\vartheta}^{\dot{\alpha}} \tilde{p}_{\dot{\alpha}\beta} \lambda^{\beta} \uparrow .$$

Antichiral sector goes directly into kernel of $\mathbf{q}$ with no further dynamical conditions.

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Equivalent cohomology

In ghost degree 0, Q-cohomology $= \ker Q$ is equivalent to $\mathbf{q}$-cohomology $= \ker \mathbf{q} / \mathrm{Im}\, \mathbf{q}$ *for the chiral sector.*

In fact one retains, for the chiral sector:

- 1-particles, $\varphi^\alpha(x)\vartheta_\alpha \left|0\right\rangle$, $\varphi^{\alpha\uparrow}(x)\vartheta_\alpha \uparrow \left|0\right\rangle$, subjected to Weyl equation;
- 2-particles, $F^{(1)}$, $F^{(2)}$ subjected to R-R fields equations.

$\delta\mathbf{q}$ in chiral theory

A deformation can generate a R or R-R field by acting on reference state $|\Omega\rangle$:

$$\delta\mathbf{q}\,|\Omega\rangle$$

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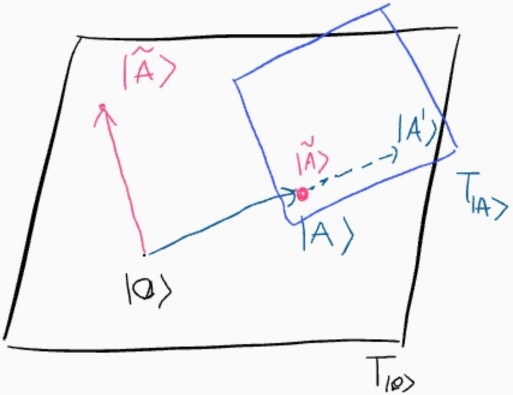
Which R-R fields can be backgrounds?

$$\delta\mathbf{q} = s\,\tilde{\vartheta}^{\dot{\alpha}}\tilde{A}^{(1)}_{\dot{\alpha}\beta}\lambda^{\beta}\uparrow, \qquad s << 1$$

$$\Downarrow$$

genuine deformation: for 1-particles it's $U(1)$ or $SU(N)$ gauge potential, for 2-particles is a 1-form background

δq in chiral theory



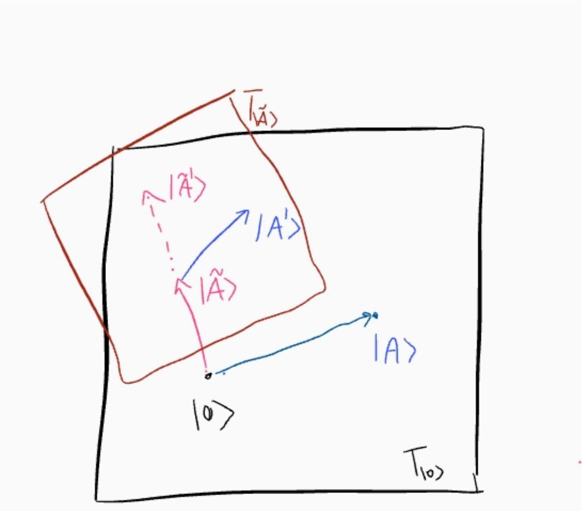
$\delta\mathbf{q}$ in chiral theory

Non-nilpotent deformations, $s << 1$:

$\delta\mathbf{q}_1 := s\,\vartheta^{\beta} A^{(1)}_{\beta\dot{\beta}} \tilde{\lambda}^{\dot{\beta}} \uparrow$

$\ker \mathbf{q}$ is in the domain of $\delta\mathbf{q}_1$.
Hence $\delta\mathbf{q}_1$ sets the antichiral sector (spinors and R-R fields) to zero.

δq in chiral theory



$\delta\mathbf{q}$ in chiral theory

Another non-nilpotent deformation:

$\delta\mathbf{q}_2 := s\,\tilde{\vartheta}^{\dot\beta}\tilde{F}^{(2)}_{\dot\beta\dot\gamma}\tilde{\lambda}^{\dot\gamma}\uparrow$

Now $\mathbf{q}+\delta\mathbf{q}_2$ lands in the antichiral sector. Perturb fields in power of s:

- for spinors:

$$\tilde{\not{p}}\varphi_{(0)} = 0, \quad \tilde{\not{p}}\varphi_{(1)} = \tilde{F}^{(2)}\tilde{\chi}_{(0)}$$

- for RR-fields: $\tilde{\not{p}}_{\dot\alpha}{}^{\alpha}A_{\alpha}{}^{\dot\beta}(x)$ is deformed by $F_{\dot\alpha\dot\gamma}F^{\dot\gamma}{}_{\dot\beta}$ interaction.

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Conclusion: can have 1-form field background $\tilde{A}^{(1)}_{\dot\alpha\beta}(x)$ and, perturbatively in s, 2-form field $\tilde{F}^{(2)}_{\dot\beta\dot\gamma}(x)$!

Summary:

- We constructed the spin fields for N=1 particle in d=4
- In the (chiral) cohomology we got R-R field strengths
- Twisting the differential by these p-forms gave us consistency conditions
- A Lagrangian from which the chiral theory can be (partially) derived exists too

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To do:

- Look into d=10 and worldsheet (ambitwistor string, pure spinor [Berkovits, Howe, 2001])
- Full Supergravity backgrounds: bosonic sector (NS-NS and R-R) and fermionic sector?

- N. Berkovits and P. S. Howe. Ten-dimensional supergravity constraints from the pure spinor formalism for the superstring. *Nucl. Phys. B*, 635:75–105, 2002. doi: 10.1016/S0550-3213(02)00352-8.
- E. Boffo and I. Sachs. Spin fields for the spinning particle. *JHEP*, 10:117, 2022. doi: 10.1007/JHEP10(2022)117.
- R. Bonezzi, A. Meyer, and I. Sachs. Einstein gravity from the $\mathcal{N} = 4$ spinning particle. *JHEP*, 10:025, 2018. doi: 10.1007/JHEP10(2018)025.
- R. Bonezzi, A. Meyer, and I. Sachs. A Worldline Theory for Supergravity. *JHEP*, 06:103, 2020. doi: 10.1007/JHEP06(2020)103.
- L. Brink and J. H. Schwarz. Quantum Superspace. *Phys. Lett. B*, 100:310–312, 1981. doi: 10.1016/0370-2693(81)90093-9.

- P. Dai, Y.-t. Huang, and W. Siegel. Worldgraph Approach to Yang-Mills Amplitudes from N=2 Spinning Particle. *JHEP*, 10:027, 2008. doi: 10.1088/1126-6708/2008/10/027.
- J. Figueroa-O'Farrill. *BRST Cohomology*, `http://www.maths.ed.ac.uk/empg/Activities/BRST`.
- D. Friedan, E. J. Martinec, and S. H. Shenker. Conformal Invariance, Supersymmetry and String Theory. *Nucl. Phys. B*, 271:93–165, 1986. doi: 10.1016/0550-3213(86)90356-1.
- D. P. Sorokin, V. I. Tkach, D. V. Volkov, and A. A. Zheltukhin. From the Superparticle Siegel Symmetry to the Spinning Particle Proper Time Supersymmetry. *Phys. Lett. B*, 216:302–306, 1989. doi: 10.1016/0370-2693(89)91119-2.