Leptogenesis: A Pedagogical Introduction*

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Abstract

This is a *preliminary* written version of a series of lectures aimed at graduate students in particle physics. We describe how the baryon asymmetry of the universe is determined from observations. We present the Sakharov conditions that are necessary for the dynamical generation of the baryon asymmetry (baryogenesis). We review the puzzle of baryogenesis, namely the failure of the Standard Model to account for the observed asymmetry. We briefly review some alternative baryogenesis scenarios. Then, we turn our focus to leptogenesis. We show that all the qualitative ingredients are guaranteed once the seesaw mechanism is assumed to be the source of neutrino masses. We reproduce the main suppression factors – related to CP violation, baryon number violation, and the departure from thermal equilibrium – that play a role in this scenario. We explain the predictive power of leptogenesis and its limitations. We describe some developments of recent years, particularly the role of (light and heavy) flavor in leptogenesis. Large parts of this write-up are based on Ref. [1].

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I. BARYOGENESIS

A. The Baryon Asymmetry of the Universe

Observations indicate that the number of baryons (protons and neutrons) in the Universe is unequal to the number of antibaryons (antiprotons and antineutrons). To the best of our understanding, all the structures that we see in the Universe – stars, galaxies, and clusters – consist of matter (baryons and electrons) and there is no antimatter (antibaryons and positrons) in appreciable quantities. Since various considerations suggest that the Universe has started from a state with equal numbers of baryons and antibaryons, the observed baryon asymmetry must have been generated dynamically, a scenario that is known by the name of *baryogenesis*.

One may wonder why we think that the baryon asymmetry has been dynamically generated, rather than being an initial condition. There are at least two reasons for that. First, if a baryon asymmetry had been an initial condition, it would have been a highly fine-tuned one. For every 6,000,000 antiquarks, there should have been 6,000,001 quarks (see Appendix A). Such a fine-tuned condition seems very implausible. Second, and perhaps more important, we have excellent reasons, based on observed features of the cosmic microwave background radiation, to think that inflation took place during the history of the Universe. Any primordial baryon asymmetry would have been exponentially diluted away by inflation.

The baryon asymmetry of the Universe poses a puzzle in particle physics. The Standard Model (SM) of particle interactions contains all the ingredients [2] that are necessary to dynamically generate such an asymmetry in an initially baryon-symmetric Universe: baryon number violation, CP violation, and departure from thermal equilibrium. Yet, it fails to explain an asymmetry as large as the one observed. New physics is called for. The new physics must, first, distinguish matter from antimatter in a more pronounced way than the weak interactions of the SM do. Second, it should provide a departure from thermal equilibrium during the history of the Universe, or modify the electroweak phase transition.

The baryon asymmetry of the Universe can be defined in two equivalent ways:

$$\eta \equiv \left.\frac{n_B - n_{\bar{B}}}{n_\gamma}\right|_0 = (6.21 \pm 0.16) \times 10^{-10}, \tag{1}$$

$$Y_{\Delta B} \equiv \left.\frac{n_B - n_{\bar{B}}}{s}\right|_0 = (8.75 \pm 0.23) \times 10^{-11} \tag{2}$$

where n_B, $n_{\bar{B}}$, n_γ and s are the number densities of, respectively, baryons, antibaryons, photons and entropy, a subscript 0 implies "at present time", and the numerical value is from combined microwave background and large scale structure data (WMAP 5 year data, Baryon Acoustic Oscillations and Type Ia Supernovae) [3]. It is convenient to calculate $Y_{\Delta B}$, the baryon asymmetry relative to the entropy density s, because $s = g_*(2\pi^2/45)T^3$ is conserved during the expansion of the Universe (g_* is the number of degrees of freedom in the plasma, and T is the temperature). The two definitions (1) and (2) are related through (see Appendix B)

$$Y_{\Delta B} = (n_{\gamma 0}/s_0)\eta \simeq \eta/7.04. \tag{3}$$

A third, related way to express the asymmetry is in terms of the baryonic fraction of the critical energy density,

$$\Omega_B \equiv \rho_B/\rho_{\rm crit} = 0.0219 \pm 0.0007. \tag{4}$$

The relation to η is given by (see Appendix B)

$$\eta = 2.74 \times 10^{-8}\ \Omega_B\ h^2, \tag{5}$$

where $h \equiv H_0/(100\ \text{km s}^{-1}\ \text{Mpc}^{-1}) = 0.701 \pm 0.013$ [4] is the present Hubble parameter.

The value of baryon asymmetry of the Universe is inferred from observations in two independent ways. The first way is via big bang nucleosynthesis [5–7]. This chapter in cosmology predicts the abundances of the light elements, D, ^{3}He, ^{4}He, and ^{7}Li. These predictions depend on essentially a single parameter which is η. The abundances of D and ^{3}He are very sensitive to η. The reason is that they are crucial in the synthesis of ^{4}He via the two body reactions D$(p,\gamma)^3$He and ^{3}He(D,p)^{4}He. The rate of these reactions is proportional to the number densities of the incoming nuclei which, in turn, depend on η: $n(\text{D}) \propto \eta$ and $n(^3\text{He}) \propto \eta^2$. Thus, the larger η, the later these ^{4}He-producing processes will stop (that is, become slower than the expansion rate of the Universe), and consequently the smaller the freeze-out abundances of D and of ^{3}He will be. The abundance of ^{4}He is less sensitive to η. Larger values of η mean that the relative number of photons, and in particular photons with energy higher than the binding energies of the relevant nuclei, is smaller, and thus the abundances of D, ^{3}He and ^{3}H build up earlier. Consequently, ^{4}He synthesis starts earlier, with a larger neutron-to-proton ratio, which results in a higher ^{4}He abundance. The dependence of the ^{7}Li-abundance on η is more complicated, as two production processes with opposite η-dependencies play a role.

The primordial abundances of the four light elements can be inferred from various observations. The fact that there is a range of η which is consistent with all four abundances gives a strong support to the standard hot big bang cosmology. This range is given (at 95% CL) by [5]

$$4.7 \times 10^{-10} \leq \eta \leq 6.5 \times 10^{-10}, \quad 0.017 \leq \Omega_B h^2 \leq 0.024. \tag{6}$$

The dependence of the various light element abundances on η is depicted in Fig. 1.

The second way to determine Ω_B is from measurements of the cosmic microwave background (CMB) anisotropies. (For pedagogical reviews, see Refs. [8, 9]; we follow here the presentation in Ref. [8].) The CMB spectrum corresponds to an excellent approximation to a blackbody radiation with a nearly constant temperature T. The basic observable is the temperature fluctuation $\Theta(\hat{\mathbf{n}}) = \Delta T/T$ ($\hat{\mathbf{n}}$ denotes the direction in the sky). The analysis is simplest in Fourier space, where we denote the wavenumber by k.

The crucial time for the CMB is that of recombination, when the temperature dropped low enough that protons and electrons could form neutral hydrogen. This happened at redshift $z_{\rm rec} \sim 1000$. Before this time, the cosmological plasma can be described, to a good approximation, as a photon-baryon fluid. The main features of the CMB follow from the basic equations of fluid mechanics applied to perfect photon-baryon fluid, neglecting dynamical effects of gravity and the baryons:

$$\ddot{\Theta} + c_s^2 k^2 \Theta = 0, \quad c_s \equiv \sqrt{\dot{p}/\dot{\rho}} = \sqrt{1/3}, \tag{7}$$

where c_s is the sound speed in the dynamically baryon-free fluid (ρ and p are the photon energy density and pressure). These features in the anisotropy spectrum are: the existence of peaks and troughs, the spacing between adjacent peaks, and the location of the first peak. The modifications due to gravity and baryons can be understood from adding their effects to Eq. (7),

$$\ddot{\Theta} + c_s^2 k^2 \Theta = F, \quad c_s = \frac{1}{\sqrt{3(1 + 3\rho_B/4\rho_\gamma)}}, \tag{8}$$

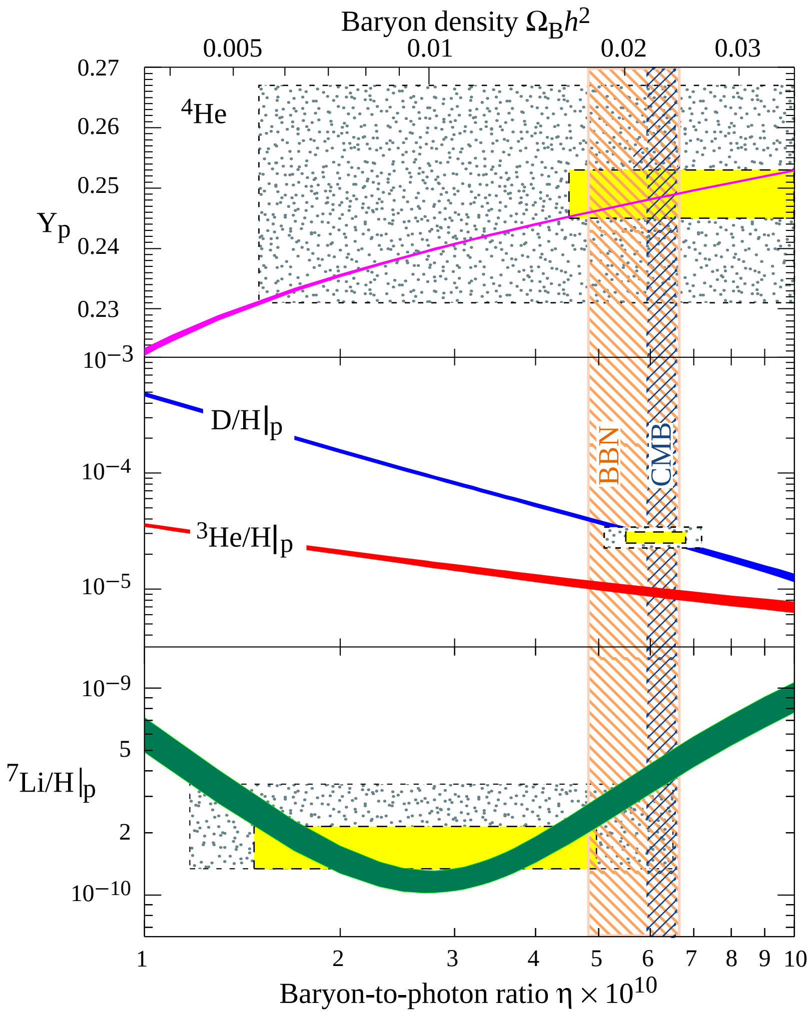

FIG. 1: The abundances of ^{4}He, D, ^{3}He, and ^{7}Li as predicted by the standard model of big bang nucleosynthesis – the bands show the 95% CL range. Boxes indicate the observed light element abundances (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematical errors). Taken from Ref. [5].

where F is the forcing term due to gravity, and ρ_B is the baryon energy density. The physical effect of the baryons is to provide extra gravity which enhances the compression into potential wells. The consequence is enhancement of the compressional phases which translates into enhancement of the odd peaks in the spectrum. Thus, a measurement of the odd/even peak disparity constrains the baryon energy density. A fit to the most recent observations (WMAP5 data only, assuming a ΛCDM model with a scale-free power spectrum for the primordial density fluctuations) gives (at 2σ) [4]

$$0.0215 \leq \Omega_B h^2 \leq 0.0240. \tag{9}$$

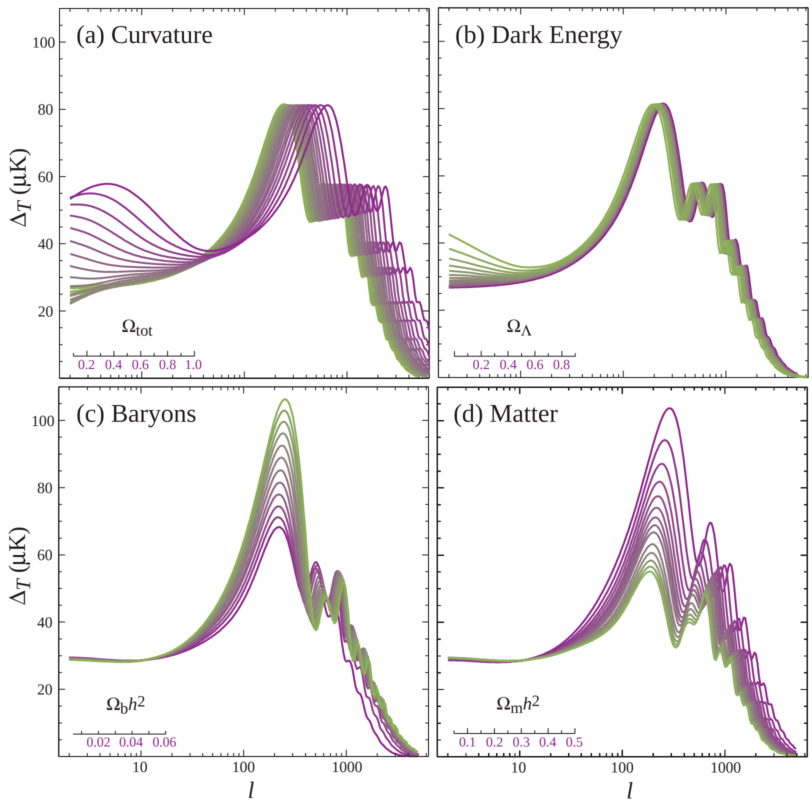

FIG. 2: Sensitivity of the acoustic spectrum to four fundamental cosmological parameters: (a) the curvature as quantified by $\Omega_{\rm tot}$; (b) the dark energy as quantified by the cosmological constant Ω_Λ ($w_\Lambda = -1$); (c) the physical baryon density $\Omega_b h^2$; (d) the physical matter density $\Omega_m h^2$, all varied around a fiducial model of $\Omega_{\rm tot} = 1,\ \Omega_\Lambda = 0.65,\ \Omega_b h^2 = 0.02,\ \Omega_m h^2 = 0.147$. Taken from Ref. [8].

The sensitivity of the CMB to various cosmological parameters is depicted in Fig. 2 [8]. In particular, the sensitivity to $\Omega_B h^2$ is demonstrated in Fig.2(c).

The impressive consistency between the nucleosynthesis (6) and CMB (9) constraints on the baryon density of the Universe is another triumph of the hot big-bang cosmology. A consistent theory of baryogenesis should explain $n_B \approx 10^{-10} s$ (and $n_{\overline{B}} = 0$).

B. Sakharov Conditions

Three conditions that are required to dynamically generate a baryon asymmetry were formulated by Sakharov [2]:

1. Baryon number violation: This condition is required in order to evolve from an initial state with $Y_{\Delta B} = 0$ to a state with $Y_{\Delta B} \neq 0$.

2. C and CP violation: If either C or CP were conserved, then processes involving baryons would proceed at precisely the same rate as the C- or CP-conjugate processes involving antibaryons, with the overall effects that no baryon asymmetry is generated.

3. Out of equilibrium dynamics: In chemical equilibrium, there are no asymmetries in quantum numbers that are not conserved (such as B, by the first condition).

These ingredients are all present in the Standard Model. However, no SM mechanism generating a large enough baryon asymmetry has been found.

1. Baryon number is violated in the Standard Model, and the resulting baryon number violating processes are fast in the early Universe [10]. The violation is due to the triangle anomaly, and leads to processes that involve nine left-handed quarks (three of each generation) and three left-handed leptons (one from each generation). A selection rule is obeyed (this selection rule implies that the sphaleron processes do not mediate proton decay):

$$\Delta B = \Delta L = \pm 3n. \tag{10}$$

At zero temperature, the amplitude of the baryon number violating processes is proportional to $e^{-8\pi^2/g^2}$ [11], which is too small to have any observable effect. At high temperatures, however, these transitions become unsuppressed [10]. (For more details, see Appendix D.)

2. The weak interactions of the SM violate C maximally and violate CP via the Kobayashi-Maskawa mechanism [12]. This CP violation can be parameterized by the Jarlskog invariant [13] which, when appropriately normalized, is of order 10^{-20} (see Appendix E). Since there are practically no kinematic enhancement factors in the thermal bath [14–16], it is impossible to generate $Y_{\Delta B} \sim 10^{-10}$ with such a small amount of CP violation. Consequently, baryogenesis implies that there must exist new sources of CP violation, beyond the Kobayashi-Maskawa phase of the Standard Model.

3. Within the Standard Model, departure from thermal equilibrium occurs at the electroweak phase transition [17, 18]. Here, the non-equilibrium condition is provided by the interactions of particles with the bubble wall, as it sweeps through the plasma. The experimental lower bound on the Higgs mass implies, however, that this transition

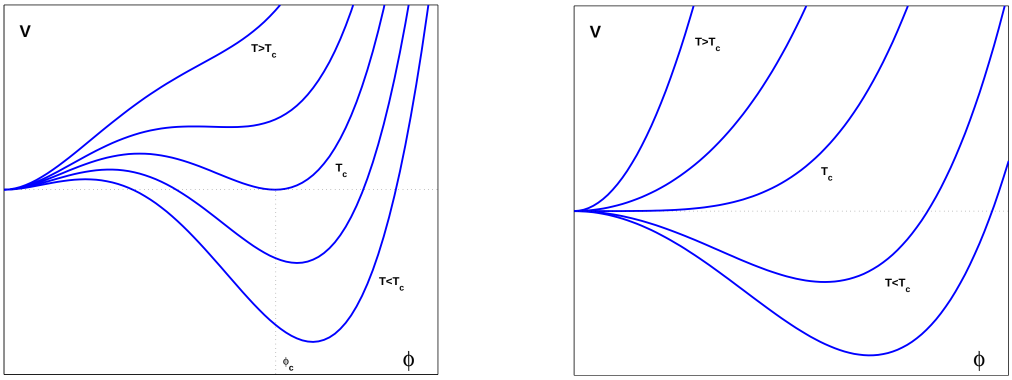

FIG. 3: The evolution of the Standard Model scalar potential with decreasing temperature for (left) $m_H < 70$ GeV and (right) $m_H > 70$ GeV.

is not strongly first order, as required for successful baryogenesis (see Fig. 3). Thus, a different kind of departure from thermal equilibrium is required from new physics or, alternatively, a modification to the electroweak phase transition.

This shows that baryogenesis requires new physics that extends the SM in at least two ways: It must introduce new sources of CP violation and it must either provide a departure from thermal equilibrium in addition to the electroweak phase transition (EWPT) or modify the EWPT itself.

C. New Physics Scenarios

Some possible new physics mechanisms for baryogenesis are the following:

GUT baryogenesis: The baryon asymmetry is generated in the out-of-equilibrium decays of heavy bosons in Grand Unified Theories (GUTs). The GUT baryogenesis scenario has difficulties with the non-observation of proton decay, which puts a lower bound on the mass of the decaying boson, and therefore on the reheat temperature after inflation. Simple inflation models do not give such a high reheat temperature which, in addition, might regenerate unwanted relics. Furthermore, in the simplest GUTs, $B+L$ is violated but $B-L$ is not. Consequently, the $B+L$ violating SM sphalerons, which are in equilibrium at $T \lesssim 10^{12}$ GeV, would destroy this asymmetry.

Leptogenesis [19]: A class of scenarios where the particle-antiparticle asymmetry is first generated in the lepton sector. In the best motivated leptogenesis models, new particles – singlet neutrinos – are introduced such that the seesaw mechanism is the source of the light neutrino masses [20]. The Yukawa couplings of the singlet neutrinos to the Standard Model Higgs and doublet-leptons provide the necessary new source of CP violation. The rate of these Yukawa interactions can be slow enough (that is slower than H, the expansion rate of the Universe, at the time that the asymmetry is generated) that departure from thermal equilibrium occurs. Lepton number violation comes from the Majorana masses of these new particles, and the Standard Model sphaleron processes play a crucial role in partially converting the lepton asymmetry into a baryon asymmetry [21].

Electroweak baryogenesis [17, 22–24]: A class of scenarios where the departure from thermal equilibrium is provided by the electroweak phase transition. In principle, the SM belongs to this class, but the phase transition is not strongly first order [25] and the CP violation is too small [14, 15]. Thus, viable models of electroweak baryogenesis need a modification of the scalar potential such that the nature of the EWPT changes, and new sources of CP violation. One example [26] is the two Higgs doublet model (2HDM), where the Higgs potential has more parameters and, unlike the SM potential, violates CP. Another interesting example is the MSSM (minimal supersymmetric SM), where a light stop modifies the Higgs potential in the required way [27, 28] and where there are new, flavour-diagonal, CP violating phases. MSSM baryogenesis requires fine tuning, but in simple extensions of the model, the fine tuning is significantly relaxed [29–31]. Electroweak baryogenesis and, in particular, MSSM baryogenesis, might soon be subject to experimental tests at the CERN LHC.

The Affleck-Dine mechanism [32, 33]: The asymmetry arises in a classical scalar field, which later decays to particles. In a SUSY model, this field could be some combination of squark, Higgs and slepton fields. The field starts with a large expectation value, and rolls towards the origin in its scalar potential. At the initial large distances from the origin, there can be contributions to the potential from baryon or lepton number violating interactions (mediated, for instance, by heavy particles). These impart a net asymmetry to the rolling field. This generic mechanism could produce an asymmetry in any combination of B and L.

Other, more exotic scenarios, are described in Ref. [34].

II. LEPTOGENESIS

A. The seesaw-leptogenesis relation

The leptonic part of the Lagrangian, when singlet fermions N_i are added, reads

$$\mathcal{L} = h^*_\beta(\bar{L}_\beta\phi^{c*})E_\beta - \lambda^*_{\alpha k}(\bar{L}_\alpha\phi^*)N_k - \frac{1}{2}\bar{N}_j M_j N^c_j + \text{h.c.}, \tag{11}$$

where $\alpha, \beta = e, \mu, \tau$. The Lagrangian terms of Eq. (11) are written in a basis where h and M are diagonal and real matrices, while λ is a generic complex matrix.

The addition of the λ and M terms, involving the N_i's, is motivated by the seesaw mechanism for light neutrino masses. When the heavy N_i's are integrated out, an effective mass matrix for the light neutrinos is generated:

$$\begin{aligned}\mathcal{L}_{m^\nu} &= \frac{1}{2}\overline{\nu^c}_\alpha m^\nu_{\alpha\beta}\nu_\beta + \text{h.c.},\\ m^\nu_{\alpha\beta} &= \lambda_{\alpha k}M_k^{-1}\lambda_{\beta k}v^2.\end{aligned} \tag{12}$$

The addition of these terms also implies, however, that the physics of the singlet fermions is likely to play a role in dynamically generating a lepton asymmetry in the Universe. The reason that leptogenesis is qualitatively almost unavoidable once the seesaw mechanism is invoked is that the Sakharov conditions are (likely to be) fulfilled:

1. Lepton number violation: The Lagrangian terms (11) violate L because lepton number cannot be consistently assigned to N_1 in the presence of λ and M. If $L(N_1) = 1$, then $\lambda_{\alpha 1}$ respects L but M_1 violates it by two units. If $L(N_1) = 0$, then M_1 respects L but $\lambda_{\alpha 1}$ violates it by one unit. (Remember that the fact that the SM interactions violate $B + L$ implies that the requirement for baryogenesis from new physics is $B - L$ violation and not necessarily B violation.)

2. CP violation: Since there are irremovable phases in λ (once h and M are chosen to be real), the Lagrangian terms (11) provide new sources of CP violation.

3. Departure from thermal equilibrium: The interactions of the N_i's are only of the Yukawa type. If the λ couplings are small enough, these interactions can be slower than the expansion rate of the Universe, in which case the singlet fermions will decay out of equilibrium.

Thus, in the presence of the seesaw terms, leptogenesis is *qualitatively* almost unavoidable, and the question of whether it can successfully explain the observed baryon asymmetry is a *quantitative* one.

We consider leptogenesis via the decays of N_1, the lightest of several (at least two) singlet neutrinos N_i. When the decay is into a single flavor α, $N_1 \to L_\alpha \phi$ or $\bar{L}_\alpha \phi^\dagger$, the baryon asymmetry can be written as follows:

$$Y_{\Delta B} \simeq \left(\frac{135\zeta(3)}{4\pi^4 g_*} \right) \times C_{\rm sphal} \times \eta \times \epsilon. \tag{13}$$

The first factor is the equilibrium N_1 number density divided by the entropy density at $T \gg M_1$. It is of $\mathcal{O}(4 \times 10^{-3})$ when the number of relativistic degrees of freedom g_* is taken as in the SM, $g_*^{\rm SM} = 106.75$ (see Appendix B for details). The other three factors on the right hand side of Eq. (13) represent the following physics aspects:

1. ϵ is the CP asymmetry in N_1 decays. For every $1/\epsilon$ N_1 decays, there is one more L than there are $\bar{L}$'s.

2. η is the efficiency factor. Inverse decays, other "washout" processes, and inefficiency in N_1 production (see below), reduce the asymmetry by $0 < \eta < 1$. In particular, $\eta = 0$ is the limit of N_1 in perfect equilibrium, so no asymmetry is generated.

3. $C_{\rm sphal}$ describes further dilution of the asymmetry due to fast processes which redistribute the asymmetry that was produced in lepton doublets among other particle species. These include gauge, Yukawa, and $B + L$ violating non-perturbative effects.

B. CP violation (ϵ)

The CP asymmetry produced in N_1 decay is defined by

$$\epsilon \equiv \frac{\Gamma(N_1 \to \phi L) - \Gamma(N_1 \to \phi^\dagger \bar{L})}{\Gamma(N_1 \to \phi L) + \Gamma(N_1 \to \phi^\dagger \bar{L})}. \tag{14}$$

It arises from the interference of tree-level (subscript 0) and one-loop (subscript 1) amplitudes. The tree and loop matrix elements can each be separated into a coupling constant part c and an amplitude part $\mathcal{A}$:

$$\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 = c_0 \mathcal{A}_0 + c_1 \mathcal{A}_1. \tag{15}$$

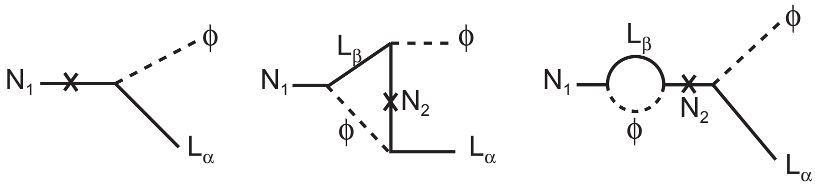

FIG. 4: The diagrams contributing to the CP asymmetry ϵ.

For example, in the tree level decay of Fig. 4,

$$c_0 = \lambda^*_{\alpha 1}, \qquad \mathcal{A}_0(N \to \phi^\dagger \bar{L}_\alpha) = \bar{u}_{L_\alpha} P_R u_N. \tag{16}$$

The matrix element for the CP conjugate process is

$$\overline{\mathcal{M}} = c_0^* \overline{\mathcal{A}}_0 + c_1^* \overline{\mathcal{A}}_1, \tag{17}$$

where in the CP conjugate amplitude the u_X spinors are replaced by v_X spinors. Since $\bar{u}_X u_X = \not{p} = \bar{v}_X v_X$, the magnitudes are the same, $|\overline{\mathcal{A}}_i|^2 = |\mathcal{A}_i|^2$. Thus the CP asymmetry can be written as

$$\begin{aligned} \epsilon &= \frac{\int |c_0 \mathcal{A}_0 + c_1 \mathcal{A}_1|^2 \tilde{\delta} d\Pi_{L,\phi} - \int |c_0^* \overline{\mathcal{A}}_0 + c_1^* \overline{\mathcal{A}}_1|^2 \tilde{\delta} d\Pi_{L,\phi}}{2 \int |c_0 \mathcal{A}_0|^2 \tilde{\delta} d\Pi_{L,\phi}} \\ &= \frac{\mathcal{I}m(c_0 c_1^*)}{|c_0|^2} \frac{2 \int \mathcal{I}m(\mathcal{A}_0 \mathcal{A}_1^*) \tilde{\delta} d\Pi_{L,\phi}}{\int |\mathcal{A}_0|^2 \tilde{\delta} d\Pi_{L,\phi}}, \end{aligned} \tag{18}$$

where

$$\tilde{\delta} = (2\pi)^4 \delta^4(P_i - P_f), \quad d\Pi_{L,\phi} = d\Pi_L d\Pi_\phi = \frac{d^3 p_L}{2E_L (2\pi)^3} \frac{d^3 p_\phi}{2E_\phi (2\pi)^3}, \tag{19}$$

and P_i, P_f are, respectively, the incoming four-momentum (in this case P_N) and the outgoing four-momentum (in this case $P_\phi + P_L$). The loop amplitude $\mathcal{A}_1$ has an imaginary part when there are branch cuts corresponding to intermediate on-shell particles, which can arise in the loops of Fig. 4 when the ϕ and L are on-shell:

$$2\mathcal{I}m(\mathcal{A}_0 \mathcal{A}_1^*) = \mathcal{A}_0(N \to \phi L) \int \mathcal{A}_0^*(N \to \bar{L}' \phi'^\dagger) \tilde{\delta}' d\Pi_{L',\phi'} \mathcal{A}_0^*(\bar{L}' \phi'^\dagger \to \phi L). \tag{20}$$

Here ϕ' and L' are the (assumed massless) intermediate on-shell particles.

In the limit $M_{i>1} \gg M_1$, the effects of $N_{i>1}$ can be represented by an effective dimension-5 operator. In the diagrams of Fig. 4, this corresponds to shrinking the $N_{i>1}$-propagator to a point. For calculating ϵ, the Feynman rule for the dimension-5 operator can be taken $\propto m^\nu / v^2$. (There is a contribution to m^ν from N_1 exchange, which is not present in the dimension-5 operator that is obtained by integrating out N_2 and N_3. But the N_1-mediated part of m^ν makes no contribution to the imaginary part for ϵ.) Then we obtain for the relevant coupling constants

$$c_0 = \lambda^*_{\alpha 1}, \quad c_1 = (3/v^2)\sum_\beta \lambda_{\beta 1} m^{\nu *}_{\beta\alpha}. \tag{21}$$

Using $\mathcal{A}^*_0(\bar{L}\phi^\dagger \to \phi L) = \bar{v}_L P_L u_L$, we obtain

$$\epsilon = \frac{3M_1}{16\pi v^2 (\lambda^\dagger\lambda)_{11}} \mathcal{I}m \left[\sum_{\alpha,\beta} \lambda^*_{\alpha 1}\lambda^*_{\beta 1} m^\nu_{\beta\alpha}\right]. \tag{22}$$

Noting that the three-vector

$$\hat{\lambda}_\alpha = \frac{\lambda^\dagger_{1\alpha}}{\sqrt{(\lambda^\dagger\lambda)_{11}}}, \tag{23}$$

is a unit vector ($\sum_\alpha |\hat{\lambda}_{1\alpha}|^2 = 1$), and using

$$U^T m^\nu U = D^\nu, \tag{24}$$

where U is the leptonic mixing matrix and $D^\nu = \mathrm{diag}(m_1, m_2, m_3)$, we have

$$\begin{aligned} \mathcal{I}m(\hat{\lambda}^T m^\nu \hat{\lambda}) &= \mathcal{I}m(\hat{\lambda}^T U^* U^T m^\nu U U^\dagger \hat{\lambda}) = \mathcal{I}m[(U^\dagger\hat{\lambda})^T D^\nu (U^\dagger\hat{\lambda})] \\ &= \mathcal{I}m(\hat{\lambda}'^T D^\nu \hat{\lambda}') = \mathcal{I}m(\sum_\alpha \hat{\lambda}'^2_\alpha m_\alpha) \leq m_{\max}, \end{aligned} \tag{25}$$

where $\hat{\lambda}' \equiv U^\dagger\hat{\lambda}$ is another unit vector, and $m_{\max}$ is the heaviest light neutrino mass. Consequently,

$$|\epsilon| \leq \frac{3M_1 m_{\max}}{16\pi v^2}. \tag{26}$$

The upper bound on $|\epsilon|$ can be used to obtain a lower bound on M_1:

$$M_1 \gtrsim 10^9 \text{ GeV} \left(\frac{m_{\rm atm}}{m_{\max}}\right)\left(\frac{|\epsilon|}{10^{-7}}\right). \tag{27}$$

The lower bound on M_1 further implies a lower bound on the reheat temperature after inflation.

One can go beyond the effective theory and incorporate the $N_{2,3}$ states as dynamical degrees of freedom. For a not-too-degenerate N_i spectrum, $M_{i>1} - M_1 \gg \Gamma_D$, one obtains

$$\epsilon = \frac{1}{8\pi}\frac{1}{(\lambda^\dagger\lambda)_{11}} \sum_j \mathcal{I}m\left\{\left[(\lambda^\dagger\lambda)_{1j}\right]^2\right\} g(x_j), \tag{28}$$

where

$$x_j \equiv M_j^2/M_1^2, \tag{29}$$

and, within the SM [35],

$$g(x) = \sqrt{x}\left[\frac{1}{1-x} + 1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right]. \tag{30}$$

Expanding in powers of M_1/M_j, one obtains for the decay into a specific flavor $\alpha = e, \mu, \tau$,

$$\epsilon_\alpha = -\frac{1}{8\pi(\lambda^\dagger\lambda)_{11}}\sum_{j\neq 1}\mathcal{I}m\left\{(\lambda\lambda^\dagger)_{j1}\left[\frac{3M_1}{2M_j}(\lambda\lambda^\dagger)_{j1} + \frac{M_1^2}{M_j^2}(\lambda\lambda^\dagger)_{1j} + \frac{5M_1^3}{6M_j^3}(\lambda\lambda^\dagger)_{j1} + \ldots\right]\right\}. \tag{31}$$

The three terms in square brackets correspond, respectively, to the dimension-five term $(\ell\phi)^2$, dimension-six term $(\bar{\ell}\phi^*)\ \not\partial(\ell\phi)$, and dimension-seven term $(\ell\phi)\partial^2(\ell\phi)$. The $d = 5$ term is the neutrino mass term. It leads to Eqs. (22) and (26) and to the Davidson-Ibarra (DI) bound on the size of the asymmetry [36]:

$$|\epsilon^{d=5}| = |\sum_\alpha \epsilon_\alpha^{d=5}| \leq \frac{3}{16\pi}\frac{M_1(m_3 - m_1)}{v^2}. \tag{32}$$

(When combined with a calculation of the washout factor, the bound leads to $m_3 \leq 0.13$ eV and $\tilde{m} \leq 0.28$ eV.) The DI bound demonastrates how subtle relations between the leptogenesis parameters and the neutrino mass parameters might arise, so we derive it in Appendix G. Yet, the DI bound can be violated when any of the following conditions holds:

1. The heavy singlet fermions are not strongly hierarchical [37–39] (the bound is not obeyed by the $d = 7$ term);

2. There are more than three singlet fermions (Appendix H);

3. There are significant contributions to the asymmetry from $N_{2,3}$ decays;

4. Flavor effects play a role [40, 41], which is the case when $T \lesssim 10^{12}$ GeV.

The $d = 6$ term does not break total lepton number. Indeed, its contribution to $\sum_\alpha \epsilon_\alpha$ vanishes. Yet, when flavor effects are important and, in particular, when the washout factors are flavor-dependent, it can generate the lepton asymmetry [42].

C. Out-of-equilibrium dynamics (η)

The non-equilibrium which is necessary for leptogenesis is provided by the expansion of the Universe: interaction rates which are of order, or slower than the Hubble expansion rate are not fast enough to equilibrate particle distributions. Interactions can be classified as much faster than H, much slower, or of the same order. For the purpose of making analytic estimates, it is convenient to have a single scale problem. The time-scale of leptogenesis is H^{-1}, so we neglect interactions that are much slower than H. Interactions that are faster than H are resummed into thermal masses, and impose chemical and kinetic equilibrium conditions on the distributions of particles whose interactions are fast. (In Appendix C we use chemical equilibrium constraints to demonstrate how thermal equilibrium prevents the generation of asymmetries.)

One can formulate a rough rule for when N_1 decays out of equilibrium, which is

$$\Gamma_D < H(T = M_1), \tag{33}$$

where Γ_D is the decay rate of N_1,

$$\Gamma_D = \frac{(\lambda^\dagger \lambda)_{11} M_1}{8\pi}. \tag{34}$$

and $H(T = M_1)$ is the expansion rate of the Universe at the time when the temperature equals the mass of N_1,

$$H(T = M_1) = 1.66 g_*^{1/2} \frac{M_1^2}{m_{\rm Pl}}. \tag{35}$$

Here g_* is the number of relativistic degrees of freedom in the thermal bath. Within the SM, $g_* = 106.75$.

It is useful to introduce two dimensionful parameters [43], $\tilde{m}$ and m_*, which are of the order of the light neutrino masses and which represent, respectively, the decay rate Γ_D and expansion rate $H(T = M_1)$:

$$\begin{aligned} \tilde{m} &\equiv \frac{8\pi v^2}{M_1^2} \Gamma_D = \frac{(\lambda^\dagger \lambda)_{11} v^2}{M_1}, \\ m_* &\equiv \frac{8\pi v^2}{M_1^2} H(T = M_1) \simeq 1.1 \times 10^{-3} \text{ eV}. \end{aligned} \tag{36}$$

It can be shown [44] that $\tilde{m} > m_{\rm min}$ where $m_{\rm min}$ is the lightest light neutrino mass (see Appendix F), so that generically $\tilde{m} \gtrsim m_{\rm sol}$ [45, 46]. The $\Gamma_D < H(T = M_1)$ condition for

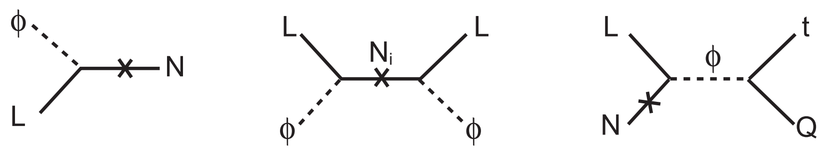

FIG. 5: Washout processes: inverse decays, $\Delta L = 2$ scattering, and $\Delta L = 1$ scattering.

out-of-equilibrium decay reads, in the language of $\tilde{m}$ and m_*, simply as

$$\tilde{m} < m_*. \tag{37}$$

If indeed $\tilde{m} \gtrsim m_{\rm sol}$, then this condition is not satisfied. This range of parameters is referred to as "strong washout". The converse, less likely, case of $\tilde{m} \lesssim m_*$, is referred to as "weak washout".

In the strong washout regime, at $T \sim M_1$, a thermal number density of N_1 is obtained ($n_{N_1} \sim n_\gamma$) independent of the initial conditions. The total lepton number asymmetry at $T \sim M_1$ is highly suppressed, $Y_L \simeq 0$: any asymmetry made in the production of N_1 is washed out. As the temperature drops down, and the N_1's start to decay, the inverse decays $L\phi \to N_1$, which can wash out the asymmetry, may initially be fast compared to H. The asymmetry will survive once the inverse decays (and lepton number violating scattering processes – see Fig. 5) are out of equilibrium:

$$\Gamma_{ID} \equiv \Gamma(\phi L \to N_1) \simeq \frac{1}{2}\Gamma_D e^{-M_1/T} < H = 1.66 g_*^{1/2}\frac{T^2}{m_{\rm Pl}}, \tag{38}$$

where Γ_D is given in Eq. (34). At temperature T_F, where Eq. (38) is satisfied, the remaining N_1 density is Boltzmann suppressed, $\propto e^{-M_1/T_F}$. Below T_F, the N_1's decay out of equilibrium and contribute to to the lepton asymmetry. So the efficiency factor η can be estimated as

$$\eta \simeq \frac{n_{N_1}(T_f)}{n_{N_1}(T \gg M_1)} \simeq e^{-M_1/T_F} \simeq \frac{m_*}{\tilde{m}}, \quad \tilde{m} > m_*, \tag{39}$$

where $m_*/\tilde{m} = H(T = M_1)/\Gamma_D$. This approximation holds for $\tilde{m} > m_* \simeq 0.001$ eV.

In the weak washout regime, the total decay rate is small, $\tilde{m} < m_*$. The final lepton asymmetry now depends on the initial conditions. The most interesting scenario is called "thermal leptogenesis": It assumes that the initial N_1 number density (at $T \gg M_1$) is zero. This is a rather unique scenario because the same interactions ($\lambda_{\alpha 1}$) are responsible for the production and the decays of N_1 [43, 47]. For $\tilde{m} < m_*$, the number density n_{N_1} does not reach the equilibrium number density. The reason is that the N_1 are produced by inverse decays, $\phi L \to N_1$, and, most effectively, by $2 \to 2$ scatterings involving the top quark, $q_L t_R \to \phi \to L N_1$ or $L t_R \to \phi \to q_L N_1$:

$$\Gamma_{\rm prod} \sim \frac{Y_t^2 (\lambda^\dagger \lambda)_{11}}{4\pi} T. \tag{40}$$

Since $Y_t \sim 1$, $\Gamma_D < H(T = M_1)$ translates into $\Gamma_{\rm prod}(T = M_1) < H(T = M_1)$. The N_1 production is most efficient at $T \sim M_1$, when the age of the Universe satisfies $\tau_U = 1/(2H)$, so $n_{N_1} \sim \Gamma_{\rm prod} \tau_U n_\gamma \sim (\tilde{m}/m_*) n_\gamma$.

Another peculiar feature of thermal leptogenesis is that the CP asymmmetry in the processes that produce the N_1 population is closely related to the CP asymmetry in the N_1 decays. In particular, for hierarchical N_i's, the CP asymmetry in the scattering interactions by which the N_1 population is produced is equal in magnitude but opposite in sign to the CP asymmetry in N_1 decays. At first sight, this suggests that the final lepton asymmetry is zero. A non-zero asymmetry survives, however, because the initial "anti-asymmetry" made with the N_1 population is depleted by scattering, decays, and inverse decays. This washout is critical to the viability of thermal leptogenesis. It introduces yet another suppression factor of order $\tilde{m}/m_*$. Thus, for weak washout and thermal leptogenesis, we have

$$\eta \sim \frac{\tilde{m}^2}{m_*^2}, \qquad \tilde{m} \ll m_*, \quad n_{N_1}(T \gg M_1) \simeq 0. \tag{41}$$

As stated above, for weak washout, the final lepton asymmetry depends on the initial conditions. If, for example, the initial N_1 population has equilibrium number density, we have $\eta \simeq 1$. If it is much larger than the equilibrium density, *i.e.* the energy density in N_1 dominates the energy density of the Universe at early times, then we have effectively $\eta > 1$ [48].

D. Lepton and $B + L$ violation ($C_{\rm sphal}$)

The N_1 decay, which depends on both M_1 and $\lambda_{\alpha 1}$, does not conserve L. The heavy mass eigenstate is its own antiparticle, so it can decay both to $L\phi$ and to $\bar{L}\phi^*$. If there is an asymmetry in the rates, a net lepton asymmetry will be produced.

The baryon number violation is provided by $B + L$ changing SM non-perturbative processes [17] (see Appendix D). Their approximate rate is given by Eq. (D14) and is faster than the Hubble expansion rate H in the thermal plasma for $T \lesssim 10^{12}$ GeV. It partially transfers an initial lepton asymmetry or, more precisely, $B-L$ asymmetry, into a final baryon asymmetry according to (for the SM)

$$Y_{\Delta B} \simeq Y_{\Delta(B-L)} \times \begin{cases} \frac{28}{79} & T > T_{\rm EWPT}, \\ \frac{12}{37} & T < T_{\rm EWPT}. \end{cases} \tag{42}$$

In this subsection, we explain how to derive the first of the relations (42).

Let us define

$$Y_{\Delta_\alpha} = \frac{1}{3} Y_{\Delta B} - Y_{\Delta L_\alpha}, \quad \alpha = e, \mu, \tau, \tag{43}$$

so that

$$Y_{\Delta(B-L)} = \sum_\alpha Y_{\Delta_\alpha}. \tag{44}$$

The important feature of Y_{Δ_α} is that, after the era of leptogenesis, the Δ_α are conserved. When the lepton number violating interections of N_1 drop out of equilibrium, the only remaining fast interactions that violate lepton and/or baryon number are the sphaleton interatcions, but these conserve the three Δ_α. We thus aim to obtain a relation between $Y_{\Delta B}$ and the Y_{Δ_α}. Such a relation depends on which other interactions are in equilibrium, and so we should calculate it at the temperature when the sphalerons go out of equilibrium.

Various SM interactions can change the number of particles of different species. For example, the Lagrangian term $h_\alpha \bar{L}_\alpha \phi^c E_\alpha$ changes a charged $SU(2)$-singlet lepton into a Higgs and an $SU(2)$-doublet lepton. If such interactions are fast, compared to the expansion rate H, they lead to an equilibrium state where the comoving number densities of the participating particles remain constant. This is described by conditions of chemical equilibrium: The sum of chemical potentials, over all particles entering the interaction, should be zero. For example, if the charged lepton Yukawa interaction is fast, we have

$$\mu_{E_\alpha} - \mu_{L_\alpha} + \mu_\phi = 0. \tag{45}$$

The set of reactions that are in chemical equilibrium enforce algebraic relations between various chemical potentials [21, 49]. In what follows, it would be convenient to define for quarks and leptons

$$\mu_X = \sum_{i=1,2,3} \mu_{X_i}. \tag{46}$$

The chemical potentials can be related to the asymmetries in particle number densities by expanding the distribution functions for small μ/T (see Appendix B):

$$\Delta y_i \equiv \frac{n_i - n_{\bar{i}}}{s} = \begin{cases} \frac{g_i}{6s} T^2 \mu_i & \text{fermions} \\ \frac{g_i}{3s} T^2 \mu_i & \text{bosons} \end{cases} \tag{47}$$

where g_i is the number of degrees of freedom of the particle:

$$g_Q = 6,\ g_U = g_D = 3,\ g_L = 2,\ g_E = 1,\ g_\phi = 2. \tag{48}$$

For T just above the EWPT, all SM interactions are fast. Fast Yukawa interactions lead to the following relations among chemical potentials:

$$\begin{aligned} \mu_{U_i} &= \mu_{Q_i} + \mu_\phi, \\ \mu_{D_i} &= \mu_{Q_i} - \mu_\phi, \\ \mu_{E_i} &= \mu_{L_i} - \mu_\phi. \end{aligned} \tag{49}$$

The condition from QCD sphaleron interactions is redundant (see *e.g.* Ref. [1]). Fast EW sphaleron interactions lead to

$$3\mu_Q + \mu_L = 0. \tag{50}$$

Hypercharge neutrality requires

$$\mu_Q + 2\mu_U - \mu_D - \mu_L - \mu_E + 2\mu_\phi = 0, \tag{51}$$

Combining Eqs. (49), (50) and (51), we obtain

$$\mu_\phi = -\frac{4}{7}\mu_Q. \tag{52}$$

The baryon and lepton number asymmetries are given in terms of particle number asymmetries by

$$\begin{aligned} Y_{\Delta L} &= Y^{\rm eq} \sum_i (\Delta y_{L_i} + \Delta y_{E_i}), \\ Y_{\Delta B} &= Y^{\rm eq} \frac{1}{3} \sum_i (\Delta y_{Q_i} + \Delta y_{U_i} + \Delta y_{D_i}). \end{aligned} \tag{53}$$

Using Eq. (47), we can express the asymmetries in terms of the chemical potentials:

$$\begin{aligned} Y_{\Delta L} &= \frac{T^2 y^{\rm eq}}{6}(g_L \mu_L + g_E \mu_E), \\ Y_{\Delta B} &= \frac{T^2 y^{\rm eq}}{18}(g_Q \mu_Q + g_U \mu_U + g_D \mu_D). \end{aligned} \tag{54}$$

Putting in the various g_X values of Eq. (48), we obtain

$$Y_{\Delta L} = -\frac{17}{14} T^2 y^{\rm eq} \mu_Q, \tag{55}$$

$$Y_{\Delta B} = \frac{2}{3} T^2 y^{\rm eq} \mu_Q,$$

$$Y_{\Delta(B-L)} = \frac{79}{42} T^2 y^{\rm eq} \mu_Q. \tag{56}$$

We finally get

$$Y_{\Delta B} = \frac{28}{79} Y_{\Delta(B-L)}. \tag{57}$$

E. Baryon asymmetry from leptogenesis

We can finally put all the ingredients that we have investigated together, and get an estimate of the baryon asymmetry from leptogenesis. Using Eq. (13) as our starting point, focussing on the strong washout regime and making the single flavor approximation, so that η is given by Eq. (39), and using Eq. (42) for $C_{\rm sphal}$, we obtain

$$Y_{\Delta B} \sim 10^{-3} \, \frac{10^{-3} \text{ eV}}{\tilde{m}} \, \epsilon, \tag{58}$$

where ϵ and $\tilde{m}$ depend on the seesaw parameters via, respectively, Eq. (28) and Eq. (36).

The plausible range for $\tilde{m}$ is the one suggested by the range of hierarchical light neutrino masses, $10^{-3} - 10^{-1}$ eV, so we expect a rather mild washout effect, $\eta \gtrsim 0.01$. Then, to account for $Y_{\Delta B} \sim 10^{-10}$, we need $|\epsilon| \gtrsim 10^{-5} - 10^{-6}$. Using Eq. (28), we learn that this condition roughly implies, for the seesaw parameters,

$$\frac{M_1}{M_2} \frac{\mathcal{I}m[(\lambda^\dagger \lambda)^2_{12}]}{(\lambda^\dagger \lambda)_{11}} \gtrsim 10^{-4} - 10^{-5}, \tag{59}$$

which is quite natural. More concretely, taking as rough estimate $\lambda^2 v^2 / M_1 \sim 10^{-2}$ eV, then $\lambda \gtrsim 10^{-2}$ is very plausible for $M_1 \gtrsim 10^{11}$ GeV.

We can thus conclude that leptogenesis is attractive not only because all the required features are qualitatively present, but also because the quantitative constraints are plausibly satisfied. In particular, $\tilde{m} \sim 0.01$ eV, as suggested by the light neutrino masses, is optimal for thermal leptogenesis as it leads to effective production of N_1's in the early Universe and only mild washout effects. Furthermore, the required CP asymmetry can be achieved in large parts of the seesaw parameter space.

III. FLAVOR

Various ingredients that affect leptogenesis and had not been taken into account in the original calculations have been identified and analyzed in recent years. These include, for example, finite temperature effects [48] and spectator processes [50, 51].

The quantitatively most significant ingredient is, however, the flavor decomposition of the singlet neutrinos. Flavor can involve the heavier singlet neutrinos in leptogenesis in new ways, and it can enhance the final baryon asymmetry quite generically by a factor of a few, and in special cases by an order of magnitude. Flavor physics brings more parameters into the leptogenesis picture and, while making the physics richer, reduces the predictive power of this scenario. In this section, we explain several simple flavor effects that are qualitatively interesting and of potential quantitative importance.

A. Light flavor

The interaction rate for a charged lepton Yukawa interaction can be estimated as

$$\Gamma_{h_\alpha} \simeq 5 \times 10^{-3} h_\alpha^2 T. \tag{60}$$

Comparing this rate to the expansion rate of the Universe,

$$H(T) = 1.66 g_*^{1/2} \frac{T^2}{m_{\rm Pl}}. \tag{61}$$

we learn that the charged lepton Yukawa interactions are fast at temperatures below

$$T \lesssim 3 \times 10^{-4} m_{\rm Pl} h_\alpha^2 \sim \begin{cases} 10^{12}\ \text{GeV} & \alpha = \tau, \\ 10^{9}\ \text{GeV} & \alpha = \mu, \\ 10^{6}\ \text{GeV} & \alpha = e. \end{cases} \tag{62}$$

(In two Higgs doublet models, such as the MSSM, the respective temperatures are higher by a factor of $(1 + \tan^2\beta)$.) Hence, above $T \sim 10^{12}$ GeV, flavor effects can be neglected, and our analysis in Section II holds. At temperatures below $T \sim 10^{12}$ GeV, the single flavor approximation fails in general, and several, potentially significant flavor effects play a role.

In this Section we do not aim to explain all possible flavor effecs. Instead, we will just explain a single, rather generic (and arguably the simplest) such effect.

When charged lepton flavor Yukawa interactions are slow, then the lepton doublet state that propagarates in spacetime is the combination of flavor eigenstates to which N_1 decays,

$$L_{N_1} = \sum_\alpha \hat{\lambda}_\alpha L_\alpha \tag{63}$$

In contrast, when the h_α interactions are faster than the expansion rate of the Universe, then the lepton doublet states that propagate in spacetime are the various flavor states L_α. We define projection operators,

$$\begin{aligned} P_\alpha &\equiv |\langle L_\alpha | L_{N_1}\rangle|^2, \\ \bar{P}_\alpha &\equiv |\langle \bar{L}_\alpha | \bar{L}_{N_1}\rangle|^2, \\ \Delta P_\alpha &\equiv P_\alpha - \bar{P}_\alpha. \end{aligned} \tag{64}$$

At tree level, $P_\alpha = \bar{P}_\alpha = |\lambda_{1\alpha}|^2$, but at the one-loop level we have, in general, $\Delta P_\alpha \neq 0$. The physical meaning of $\Delta P_\alpha \neq 0$ is that the lepton and the antilepton states to which N_1 decays are not CP-conjugates of each other, which is a qualitatively new source of CP asymmetry that is not present in the unflavored case. We will, however, ignore this effect in what follows.

Putting by hand $\Delta P_\alpha = 0$, the expressions for the CP asymmetry and the washout parameter for a specific flavor are simply given by

$$\epsilon_\alpha = P_\alpha \epsilon, \quad \tilde{m}_\alpha = P_\alpha \tilde{m}. \tag{65}$$

In case that the individual lepton flavors are resolved by fast Yukawa interactions, the expression of Eq. (13) for the baryon asymmetry is replaced by

$$Y_{\Delta B} \simeq \left(\frac{135\zeta(3)C_{\rm sphal}}{4\pi^4 g_*}\right) \sum_\alpha \eta_\alpha \, \epsilon_\alpha. \tag{66}$$

The fact that some of the flavors could be in the weak washout regime while others are in the strong washout regime opens the way to interesting effects that often have dramatic quantitative consequences.

We again choose to focus on the simplest scenario. We take all P_α to be approximately equal to each other,

$$P_\alpha \simeq \frac{1}{\mathcal{N}_f}, \tag{67}$$

where $\mathcal{N}_f$ is the number of flavors that are resolved by fast charged lepton Yukawa interactions: $\mathcal{N}_f = 2(3)$ for $T \in 10^9$ GeV – 10^{12} GeV ($< 10^9$ GeV). We further assume that $\tilde{m}_\alpha > m_*$ for all flavors. Then

$$\sum_\alpha \eta_\alpha\ \epsilon_\alpha = \sum_\alpha \frac{\eta}{P_\alpha}\ (P_\alpha \epsilon) = \sum_\alpha \eta\ \epsilon \simeq \mathcal{N}_f\ \eta\ \epsilon. \tag{68}$$

We learn that in a generic case, where the lepton-doublet state to which N_1 decays is neither aligned nor orthogonal to any of the flavor directions e, μ, τ, the final baryon asymmetry will be enhanced by a factor of 2–3 if leptogenesis occurs at temperatures lower than about 10^{12} GeV.

Some of the early literature on flavor effects in leptogeneis, where additional consequences are discussed, includes Refs. [40, 41, 52–54].

The phenomenological consequences are as follows:

- In the single flavor approximation, the DI bound leads to an upper bound of order 0.15 eV on the light neutrino masses. Flavor effects lift the bound (with some tuning of parameters) to the eV scale, where direct bounds apply anyway.

- An important, but disappointing, feature of the single flavor approximation is the lack of model independent relation between CP violation in the leptogenesis processes and the observable phases of the lepton mixing matrix U. This remains true in the flavored case.

- Flavor effects hardly relax the lower bound on M_1 (and, consequently, on $T_{\rm reheat}$) which remains at
$$M_1 \gtrsim 2 \times 10^9\ \text{GeV} \quad (\text{non} - \text{degenerate}\ m_i). \tag{69}$$

B. Heavy flavor

In the conventional leptogenesis picture, three singlet neutrinos N_i are added to the SM, with hierarchical Majorana masses, $M_1 \ll M_1 \ll M_3$. It is often assumed that the L-violating effects of N_1 would washout any lepton asymmetry $\Delta y_{L_{N_{2,3}}}$ generated at temperatures $T \gg M_1$ in the decays of $N_{2,3}$. If this were the case, the final asymmetry would depend only on N_1 dynamics, and the number of parameters would be reduced to just M_1, $\tilde{m}$ and ϵ. However, under various, rather generic, circumstances, the lepton asymmetry generated in $N_{2,3}$ decays survives the N_1 leptogenesis phase. Thus, it is quite possible that the lepton asymmetry relevant for baryogenesis originates mainly (or, at least, in a non-negligible part) from $N_{2,3}$ decays.

The possibility that N_2 leptogenesis can successfully explain the baryon asymmetry of the Universe has been shown in two limiting cases:

1. The N_1-decoupling scenario, in which the $\lambda_{\alpha 1}$ couplings are simply too weak to washout the N_2-generated asymmetry [55–57].

2. The strong N_1-coupling scenario, where N_1-related decoherence effects project part of the lepton asymmetry from N_2 decays onto a flavor direction that is protected against N_1 washout [52, 58–60].

The N_1-decoupling scenario is simple to understand. It applies when N_1 is weakly coupled to the lepton doublets, $\tilde{m}_1 \ll m_*$. (In this subsection, we call the washout parameter $\tilde{m}$ by $\tilde{m}_1$, to emphasize that it relates to N_1 interactions, and to distinguish it from the analogously defined $\tilde{m}_2$.) In this case, the asymmetry generated in thermal N_1 leptogenesis is too small. Furthermore, the N_1 washout effects are negligible and, consequently, the asymmetry generated in N_2 decays survives.

The strong N_1-coupling scenario for N_2 leptogenesis is more subtle. For simplicity, we assume that the N_2-related washout is not too strong, while the N_1-related washout is so strong that it makes N_1 leptogenesis fail:

$$\tilde{m}_2 \not\gg m_*, \quad \tilde{m}_1 \gg m_*. \tag{70}$$

To further simplify the analysis, we impose two additional constraints: thermal leptogenesis, and strong hierarchy, $M_2/M_1 \gg 1$. Then it is guaranteed that

$$n_{N_1}(T \sim M_2) \approx 0, \quad n_{N_2}(T \sim M_1) \approx 0, \tag{71}$$

and the dynamics of N_2 and N_1 decouple.

The N_2 decays into a combination of lepton doublets that we denote by L_2:

$$|L_i\rangle = (\lambda^\dagger \lambda)_{ii}^{-1/2} \sum_\alpha \lambda_{\alpha i} |L_\alpha\rangle. \tag{72}$$

The second condition in (70) implies that already at $T \gtrsim M_1$ the N_1-Yukawa interactions are sufficiently fast to quickly destroy the coherence of L_2. Then a statistical mixture of L_1

and the state orthogonal to L_1, which we call L_0, builds up. Ignoring light flavor effects (which is appropriate if $T \gtrsim 10^{12}$ GeV), we are led to choose an orthogonal basis for the lepton doublets, (L_1, L_0, L_0'), where $\langle L_0'|L_2\rangle = 0$. Then the asymmetry ΔY_{L_2} produced in N_2 decays decomposes into two components:

$$\Delta Y_{L_0} = c^2 \Delta Y_{L_2}, \quad \Delta Y_{L_1} = s^2 \Delta Y_{L_2}, \tag{73}$$

where $c^2 \equiv |\langle L_0|L_2\rangle|^2$ and $s^2 = 1 - c^2$. The crucial point is that we expect, in general, $c^2 \neq 0$, and, since $\langle L_0|L_1\rangle = 0$, ΔY_{L_0} is protected against N_1 washout. Consequently, a finite part of the asymmetry ΔY_{L_2} from N_2 decays survives through N_1 leptogenesis. A more detailed analysis [59] finds that ΔY_{L_1} is not entirely washed out, and the final lepton asymmetry is given by $Y_{\Delta L} = (3/2)\Delta Y_{L_0} = (3/2)c^2 \Delta Y_{L_2}$.

The conclusion is that $N_{2,3}$ leptogenesis cannot be ignored, unless at least one of the following conditions applies:

1. The asymmetries and/or the washout factors vanish, $\epsilon_{N_2}\eta_2 \approx 0$ and $\epsilon_{N_3}\eta_3 \approx 0$.

2. N_1-related washout is still significant at $T \lesssim 10^9$ GeV.

3. The reheat temperature is below M_2.

IV. CONCLUSIONS

There is convincing evidence from solar, atmospheric, reactor and accelerator neutrino experiments that the SM neutrinos are massive. The seesaw mechanism extends the SM in a way that allows neutrino masses and nicely explains their lightness. Furthrmore, without any modification or addition, the physics of the seesaw mechanism – heavy Majorana fermions with Yukawa couplings to the SM lepton doublets – can also account for the observed baryon asymmetry of the Universe. The possibility of giving an explanation of two apparently unrelated experimental facts – neutrino masses and the baryon asymmetry – within a single framework that is a natural extension of the Standard Model, together with the remarkable 'coincidence' that the same neutrino mass scale suggested by neutrino oscillation data is also optimal for leptogenesis, makes the idea that baryogenesis occurs through leptogenesis a very attractive one.

Leptogenesis can be quantitatively successful without any fine-tuning of the seesaw parameters. Yet, in the non-supersymmetric seesaw framework, a fine-tuning problem arises due to the large corrections to the mass-squared parameter of the Higgs potential that are proportional to the heavy Majorana neutrino masses. Supersymmetry can cure this problem, avoiding the necessity of fine tuning. However, the gravitino problem that arises in many supersymmetric models requires a low reheat temperature after inflation, in conflict with generic leptogenesis models. Thus, constructing a fully satisfactory theoretical framework that implements leptogenesis within the seesaw framework is not a straightforward task.

From the experimental side, the obvious question to ask is if it is possible to test whether the baryon asymmetry has been really produced through leptogenesis. Unfortunately it seems impossible that any direct test can be performed. To establish leptogenesis experimentally, we need to produce the heavy Majorana neutrinos and measure the CP asymmetry in their decays. However, in the most natural seesaw scenarios, these states are simply too heavy to be produced, while if they are light, then their Yukawa couplings must be very tiny, again preventing any chance of direct measurements.

The possibility of indirect tests from low-energy measurements depends on the existence of new physics at or below the TeV scale that carries the imprint of the seesaw parameters. A plausible scenario is that of supersymmetry with flavor-universal soft supersymmetry breaking terms. Indeed, a determination of all seesaw parameters from processes involvong the supersymmrtic particles is possible in principle, though not in practice [61, 62].

Lacking the possibility of a direct proof, experiments can still provide circumstantial evidence in support of leptogenesis by establishing that (some of) the Sakharov conditions for leptogenesis are realized in nature. Planned neutrinoless double beta decay ($0\nu\beta\beta$) experiments aim at a sensitivity to the effective $0\nu\beta\beta$ neutrino mass in the few $\times$ 10 meV range. If they succeed in establishing the Majorana nature of the light neutrinos (this is likely to happen if neutrinos are quasi-degenerate or if the mass hierarchy is inverted), this will strengthen our confidence that the seesaw mechanism is at the origin of the neutrino masses and, most importantly, will establish that the first Sakharov condition for the dynamical generation of a lepton asymmetry, that is that lepton number is violated in nature, is satisfied. Proposed SuperBeam facilities and second generation off-axis SuperBeams experiments can discover CP violation in the leptonic sector. These experiments can only probe the Dirac phase of the neutrino mixing matrix. They cannot probe the Majorana low energy or the high energy phases, but the important point is that they can establish that the second Sakharov condition for the dynamical generation of a lepton asymmetry is satisfied. In contrast to the previous two conditions, verifying that the decays of the heavy neutrinos occurred out of thermal equilibrium (the third condition) remains out of experimental reach, since it would require measuring the heavy neutrino masses and the size of their couplings.

Finally, the CERN LHC has the capability of providing information that is relevant to leptogenesis. In particular, electroweak baryogenesis can be tested at the LHC. It will become strongly disfavored (if not completely ruled out) if supersymmetry is not found, or if supersymmetry is discovered but the stop and/or the Higgs are too heavy. Eliminating various scenarios that are able to explain the baryon asymmetry will strengthen the case for the remaining viable possibilities, including leptogenesis. Conversely, if electroweak baryogenesis is established by the LHC (and EDM) experiments, the case for leptogenesis will become weaker.

To conclude, the seesaw framework provides the most natural and straightforward explanation of the light neutrino masses and has, in principle, all the ingredients that are necessary for successful leptogenesis. This makes leptogenesis arguably the most attractive explanation for the observed baryon asymmetry. This scenario has limited predictive power for low energy observables, so it is unlikely to be directly tested. Yet, future experiments have the potential of strengthening, or weakening, or even falsifying the case for leptogenesis.

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APPENDIX A: THE BARYON ASYMMETRY AS AN INITIAL CONDITION

The present number density is observed to be

$$Y^0_{\Delta B} = \left(\frac{n_b - n_{\bar{b}}}{s}\right)_0 \approx 8.7 \times 10^{-11}. \tag{A1}$$

We would like to understand what kind of fine-tuning it takes to explain it by an initial condition (say, at $T \sim M_P$), namely

$$Y^i_{\Delta B} \neq 0, \tag{A2}$$

assuming only standard model physics.

For temperatures below the EWPT, $Y_{\Delta B}$ is fixed. For temperatures above the EWPT, the denominator of $Y_{\Delta B}$, that is s, is changing only by the expansion of the Universe, while the numerator, namely $n_b - n_{\bar{b}}$, is changing by both the expansion of the Universe and the sphaleron interactions. The change due to the expansion thus cancels out in $Y_{\Delta B}$, and we have

$$Y^0_{\Delta B} = C_{\rm sphal} Y^i_{\Delta(B-L)}, \tag{A3}$$

where

$$C^{\rm SM}_{\rm sphal} = \frac{28}{79}. \tag{A4}$$

We further use (for $T \gg 100$ GeV)

$$s = 1.8 g_* n_\gamma, \quad n_q \simeq n_{\bar{q}} = (7/8) n_\gamma, \tag{A5}$$

where $g^{\rm SM}_* = 106.75$. Note that $n_b = n_q/3$.

We obtain:

$$Y^0_{\Delta B} = \frac{C_{\rm sphal}}{3} \left(\frac{n_q - n_{\bar{q}}}{1.8 g_* (8/7) n_q}\right)_i, \tag{A6}$$

leading to

$$\left(\frac{n_q - n_{\bar{q}}}{n_q}\right)_i = 6.2 Y^0_{\Delta B} \frac{g_*}{C_{\rm sphal}} = 5.4 \times 10^{-10} \frac{g_*}{C_{\rm sphal}} = 1.6 \times 10^{-7}. \tag{A7}$$

Thus, we need to have about six million and one quarks for every six million quarks.

APPENDIX B: USEFUL EQUATIONS

The photon number density is given by

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3 \simeq 0.2436 T^3. \tag{B1}$$

The entropy density is given by

$$\begin{aligned} s &= \frac{2\pi^2}{45} q T^3, \\ q &= \sum_{\text{bosons}} g_B \left(\frac{T_B}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_F \left(\frac{T_F}{T}\right)^3, \\ q_0 &= 2 + 3 \times 2 \times \frac{7}{8} \left(\frac{T_\nu}{T}\right)_0^3 = 2 + \frac{21}{4}\frac{4}{11} = \frac{43}{11}. \end{aligned} \tag{B2}$$

The present ratio of photon number density to the entropy density is given by

$$\frac{s_0}{n_{\gamma 0}} = \frac{2\pi^2}{45} \times \frac{43}{11} \times \frac{\pi^2}{2\zeta(3)} \simeq 7.04 \implies \eta \simeq 7.04 Y_{\Delta B}. \tag{B3}$$

The critical energy density is given by

$$\rho_c = \frac{3H^2}{8\pi G_N} = 1.05 \times 10^{-5}\ h^2\ \text{GeV cm}^{-3}. \tag{B4}$$

Then, η and Ω_b are related as follows:

$$\eta = \frac{n_b}{n_\gamma} = \frac{\Omega_b \rho_c}{n_\gamma m_b} = \Omega_b h^2 \frac{1.05 \times 10^{-5}}{411 \times 0.938} = 2.74 \times 10^{-8}\ \Omega_b h^2. \tag{B5}$$

The equilibrium number density of massless fermions is given by

$$n_i^{\text{eq}} = \frac{3\zeta(3) g_i T^3}{4\pi^2}, \tag{B6}$$

where g_i is the number of internal degrees of freedom of the particle. For the N_i's, $g_N = 2$ because they are Majorana fermions, and

$$\frac{n_{N_1}^{\text{eq}}}{s} = \frac{135\zeta(3)}{4\pi^4 q}. \tag{B7}$$

The equlibrium number densities, such as Eq. (B6) for fermions, are derived by integrating over the Fermi-Dirac (+) or Bose-Einstein (−) distributions,

$$f_{i,\pm}^{\text{eq}}(p) = \frac{1}{e^{(E_i - \mu_i)/T} \pm 1}, \tag{B8}$$

which, expanding to leading order in the chemical potential μ_i/T and neglecting effects of order m_i/T, gives

$$n_{i,\pm}^{\text{eq}} = \frac{g_i}{(2\pi)^3} \int d^3p f_{i,\pm}^{\text{eq}}(p) = \frac{g_i T^3}{\pi^2} \times \begin{cases} \zeta(3) + \frac{\mu_i}{T}\zeta(2) & \text{(bosons)}, \\ \frac{3}{4}\zeta(3) + \frac{1}{2}\frac{\mu_i}{T}\zeta(2) & \text{(fermions)}. \end{cases} \tag{B9}$$

Eq. (47) can be derived straightforwardly from Eqs. (B9).

APPENDIX C: THE THIRD SAKHAROV CONDITION

This section is based on a lecture given by E. Nardi at the νTheME theory Institute, CERN 2010.

We use the chemical equilibrium constraints imposed by fast interactions to demonstrate explicitly that no asymmetry can be generated in thermal equilibrium. For simplicity, we work in a single generation framework. The relation between chemical potential and particle asymmetries was presented in Section II D. For a single generation, there are six chemical potentials:

$$\mu_Q,\ \mu_U,\ \mu_D,\ \mu_L,\ \mu_E,\ \mu_\phi. \tag{C1}$$

(Since N is a Majorana fermion, its chemical potential vanishes, $\mu_N = 0$.) Let us assume that all of the SM interactions are faster than the expansion rate of the Universe. Then, the SM Yukawa interactions and the EW sphaleron interactions impose four relations among the chemical potentials:

$$\begin{aligned}
\mu_Q - \mu_U + \mu_\phi &= 0,\\
\mu_Q - \mu_D - \mu_\phi &= 0,\\
\mu_L - \mu_E - \mu_\phi &= 0,\\
3\mu_Q + \mu_L &= 0.
\end{aligned} \tag{C2}$$

Hypercharge neutrality imposes an additional constraint:

$$\mu_Q + 2\mu_U - \mu_D - \mu_L - \mu_E + 2\mu_\phi = 0. \tag{C3}$$

If the neutrino Yukawa interaction is also fast enough to impose chemical equilibrium, we have a sixth relation,

$$\mu_L + \mu_\phi = 0. \tag{C4}$$

With six such conditions for six chemical potentials, the solution is

$$\mu_Q = \mu_U = \mu_D = \mu_L = \mu_E = \mu_\phi = \mu_N = 0, \tag{C5}$$

showing that no particle asymmetry is generated. If, on the other hand, the singlet neutrino is out of equlibrium, the condition (C4) is relaxed, allowing for non-vanishing asymmetries.

If any of the SM interactions is slow, so that the related condition on the chemical potentials is relaxed, a corresponding conservation law arises [63]. Explicitly, slow electron Yukawa interaction ($h_e = 0$) leads to electron number conservation,

$$\Delta n_e = 0, \tag{C6}$$

slow EW sphaleron interaction leads to baryon number conservation,

$$\Delta B = 0, \tag{C7}$$

and slow up Yukawa interaction ($h_u = 0$) implies that the (otherwise redundant) condition for fast QCD sphaleron interaction should be imposed,

$$\mu_Q - \mu_U - \mu_D = 0. \tag{C8}$$

Thus, we are always left with five conditions for six chemical potentials, implying that one chemical potential (μ_L) is sufficient to describe all asymmetries.

APPENDIX D: $B + L$ VIOLATION

This section is based on a lecture given by V. Rubakov at the Lake Louise Winter Institute, 2008.

The renormalizable Lagrangian of the Standard Model conserves the baryon number B and the three lepton flavor numbers L_α. However, due to the chiral anomaly, there are non-perturbative gauge field configurations that violate $B + L$ [11, 64, 65] (where $L = L_e + L_\mu + L_\tau$). In the early Universe, at temperatures above the electroweak phase transition (EWPT), such configurations – commonly called "sphalerons" – occur frequently and lead to rapid $B + L$ violation.

1. The chiral anomaly

Consider the Lagrangian for a massles Dirac fermion ϕ with $U(1)$ gauge interactions:

$$\mathcal{L} = \overline{\psi}\gamma^\mu(\partial_\mu - iA_\mu)\psi - \frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu}. \tag{D1}$$

It is invariant under the local symmetry

$$\psi(x) \to e^{i\theta(x)}\psi(x), \quad A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x). \tag{D2}$$

It is also invariant under a global "chiral" symmetry:

$$\psi(x) \to e^{i\gamma_5\phi}\psi(x). \tag{D3}$$

The associated current,

$$j_5^\mu = \overline{\psi}\gamma_5\gamma^\mu\psi, \tag{D4}$$

is conserved at tree level, but not in quantum theory. This can be related to the regularization of loops – renormalization introduces a scale, and the scale breaks the chiral symmetry, as would a fermion mass. At one loop, one finds

$$\partial_\mu j_5^\mu = \frac{1}{16\pi^2}\widetilde{F}_{\mu\nu}F^{\mu\nu} = \frac{\epsilon_{\rho\sigma\mu\nu}}{16\pi^2}F^{\rho\sigma}F_{\mu\nu}. \tag{D5}$$

The right-hand side can be written as a total divergence involving gauge fields, and is related to their topology: it counts the "winding number", or Chern-Simons number, of the field configuration. In four dimensions, the space-time integral of the right-hand side vanishes for an Abelian gauge field, but can be non-zero for non-Abelian fields.

In the context of leptogenesis, we are interested in the anomaly of the $B+L$ current. It arises due to the $SU(2)$ gauge interactions, which are chiral and non-Abelian. The relevant fermions are the three generations of quark and lepton $SU(2)$-doublets: $\{\psi_L^i\} = \{q_L^{a,\alpha}, \ell_L^\alpha\}$, where a is a color index and α is a generation index. The Lagrangian terms for the $SU(2)$ gauge interactions read

$$\mathcal{L} = \sum_i \overline{\psi_L}^i \left(\partial_\mu - i\frac{g}{2}\sigma^A W_\mu^A\right)\psi_L^i, \tag{D6}$$

where A is an $SU(2)$ index. The Lagrangian terms (D6) have twelve global $U(1)$ symmetries, one for each field:

$$\psi_L^i(x) \to e^{i\beta}\psi_L^i(x). \tag{D7}$$

The chiral currents associated with these transformations,

$$j_\mu^i = \overline{\psi_L}^i\gamma_\mu\psi_L^i, \tag{D8}$$

are conserved at tree level, but are anomalous in the quantum theory:

$$\partial^\mu j_\mu^i = \frac{1}{64\pi^2}F_{\mu\nu}^A\widetilde{F}^{\mu\nu A}. \tag{D9}$$

Let us define $Q^i(t) = \int j_0^i d^3x$, $\Delta Q^i = Q^i(+\infty) - Q^i(-\infty)$, and let us suppose for the moment that there exist field configurations for which

$$\Delta Q^i = \frac{1}{64\pi^2}\int d^4x\; F_{\mu\nu}^A\widetilde{F}^{\mu\nu A} \tag{D10}$$

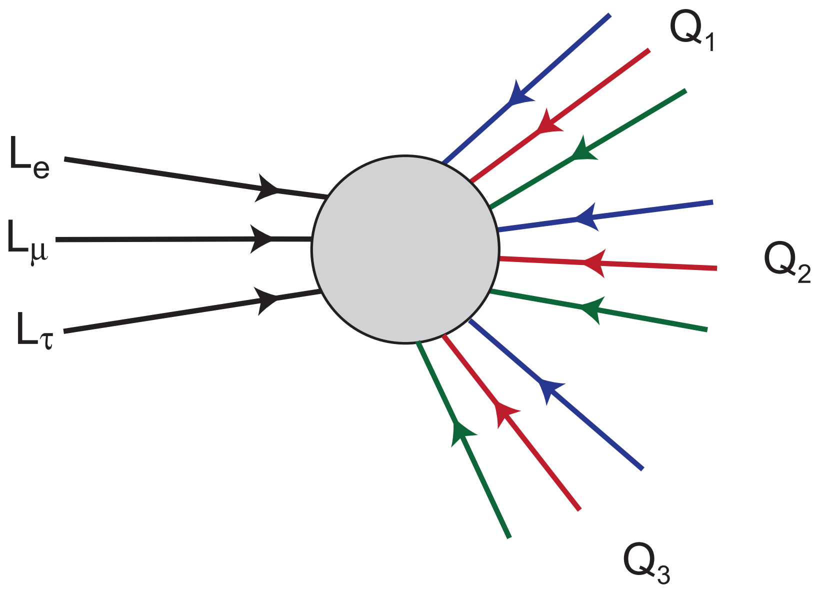

FIG. 6: A $B + L$ violating process due to non-perturbative SM effects.

is a non-zero integer. This implies that fermions will be created, even though there is no perturbative interaction in the Lagrangian that generates them.

2. $B + L$ violating rates

At zero temperature, gauge field configurations that give non-zero $\int d^4x\, \widetilde{F}F$ correspond to tunneling configurations and are called instantons. They change fermion number by an integer N, so that the instanton action is large:

$$\left| \frac{1}{4g^2} \int d^4x\, F^A_{\mu\nu} \widetilde{F}^{\mu\nu A} \right| \geq \frac{64\pi^2 N}{4g^2}. \tag{D11}$$

Consequently, the associated rate is highly suppressed,

$$\Gamma \propto e^{-(\text{instanton action})} \sim e^{-4\pi/\alpha_W}, \tag{D12}$$

and the mediated $B + L$ violation is unobservably small. Moreover, the instantons do not lead to proton decay, because an instanton acts as a source for three leptons (one from each generation), and nine quarks (all colors and generations), so it induces $\Delta B = \Delta L = 3$ processes of the type depicted in Fig. 6. The three quantum numbers $B/3 - L_\alpha$ are not anomalous, so they are conserved in the SM.

If the ground state of the gauge fields is pictured as a periodic potential, with minima labeled by integers, then the instantons correspond to vacuum fluctuations that tunnel between minima. With this analogy, one can imagine that at finite temperature, a thermal fluctuation of the field can climb over the barrier. The sphaleron is such a configuration, in the presence of the Higgs vacuum expectation value. The $B+L$ violating rate mediated by sphalerons is Boltzmann suppressed:

$$\Gamma_{\rm sph} \propto e^{-E_{\rm sph}/T} = e^{-(2Bm_W/\alpha_W)/T}, \tag{D13}$$

where $E_{\rm sph}$ is the height of the barrier at $T=0$, and $1.5 \lesssim B \lesssim 2.75$ is a parameter that depends on the Higgs mass.

For leptogenesis, we are interested in the $B+L$ violating rate at temperatures far above the EWPT. The large $B+L$ violating gauge field configurations occur frequently at $T \gg m_W$. Their rate can be estimated as [66]

$$\Gamma_{B+L \ \rm violation} \simeq 250\alpha_W^5 T. \tag{D14}$$

For temperatures below 10^{12} GeV and above the EWPT, $B+L$ violating rates are in equilibrium.

APPENDIX E: THE SUPPRESSION OF KM BARYOGENESIS

The three generation Standard Model can violate CP [12]. However, in order that CP is indeed violated, the parameters of the model must fulfill a long list of conditions:

- The up sector masses (m_u, m_c, m_t) have to be different from each other and, similarly, the down sector masses (m_d, m_s, m_b) have to be different from each other.
- The three CKM mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$ have to be different from 0 and from $\pi/2$.
- The KM phase γ has to be different from 0 and from π.

This set of conditions can be mathematically expressed as the requirement that [13] ($s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$):

$$J_{CP} \equiv (m_t^2-m_c^2)(m_t^2-m_u^2)(m_c^2-m_u^2)(m_b^2-m_s^2)(m_b^2-m_d^2)(m_s^2-m_d^2)s_{12}s_{23}s_{13}c_{12}c_{13}c_{23}s_\gamma \neq 0. \tag{E1}$$

The baryon asymmetry of the Universe is a CP violating observable. As such, it is actually proportional to J_{CP}. Note that J_{CP} has dimension of [mass^{12}]. What appears in the contribution of the KM mechanism to the baryon asymmetry is in fact a dimensionless quantity, J_{CP}/T_c^{12}, where $T_c \sim 100$ GeV is the critical temperature of the EWPT. When one puts the measured values of the quark masses and CKM parameters in J_{CP}, one obtains that $J_{CP}/T_c^{12} \sim 10^{-20}$, and thus the KM mechanism cannot explain an asymmetry as large as $\mathcal{O}(10^{-10})$.

One may wonder why the suppression by J_{CP} does not apply to all CP asymmetries measured in experiments. After all, there are CP asymmetries such as $S_{\psi K}$ that are experimentally of order one and theoretically known to be suppressed by the KM phase ($\sin 2\beta$) but by none of the mixing angles or small quark mass-squared differences of J_{CP}. The answer provides some insights as to how the KM mechanism operates. As concerns the mixing angles, they often cancel in the CP asymmetries which are ratios of CP violating to CP conserving rates. The physics behind the mass factors in Eq. (E1) is that, in order to exhibit CP violation, a process has to "go through" all three generations of quark flavors, and "sense" that their masses are different from each other. Sometimes, the experiment does that for us. For example, when experimenters measure the CP asymmetry in $B \to \psi K_S$, they already distinguish the bottom, charm, and strange masses from the others (by identifying, respectively, the B, ψ and K mass eigenstates) and thus 'get rid' of the corresponding mass factors. In contrast, baryogenesis is a flavor-blind process (it sums over all flavors), and thus suppressed by all six mass-related factors of Eq. (E1).

APPENDIX F: THE CASAS-IBARRA PARAMETRIZATION [67]

The see-saw relation is given by

$$m^\nu = v^2 \lambda^T M^{-1} \lambda. \tag{F1}$$

Using the leptonic mixing matrix to rotate to the mass basis, we have

$$D_\nu = v^2 U^T \lambda^T M^{-1} \lambda U = v^2 U^T \lambda^T M^{-1/2} M^{-1/2} \lambda U. \tag{F2}$$

Multiplying both sides on the left and on the right by $(D_\nu)^{-1/2}$ we get

$$1 = (v M^{-1/2} \lambda U D_\nu^{-1/2})^T (v M^{-1/2} \lambda U D_\nu^{-1/2}). \tag{F3}$$

The solution to this equation is

$$R = vM^{-1/2}\lambda U D_\nu^{-1/2}, \tag{F4}$$

with the condition that $R^T R = 1$. Solving for λ, we find the Casas-Ibarra parametrization:

$$\lambda = \frac{1}{v} M^{1/2} R D_\nu^{1/2} U^\dagger. \tag{F5}$$

Writing the CP asymmetry in N_1 decay in terms of the Casas-Ibarra parametrization, we obtain

$$\begin{aligned} \epsilon &= -\sum_{j=2,3} \frac{3}{16\pi} \frac{M_1}{M_j} \frac{\mathcal{I}m\left[(\lambda^\dagger\lambda)^2_{1j}\right]}{(\lambda^\dagger\lambda)_{11}} \\ &= -\frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i m_i^2 \mathcal{I}m(R_{1i}^2)}{\sum_i m_i |R_{1i}|^2}. \end{aligned} \tag{F6}$$

When the matrix R is square then $R^{-1} = R^T$. In this case, the relation $RR^T = 1$ also holds. In particular $\sum_i R_{1i}^2 = 1$.

The Casas-Ibarra parametrization provides also insights concerning the washout effects. Define a matrix [46]

$$\tilde{M} = v^2 M^{-1/2} \lambda\lambda^\dagger M^{-1/2}. \tag{F7}$$

Then, in terms of the Casas-Ibarra parametrization, we have

$$\tilde{M}_{\alpha\beta} = \sum_i m_i R_{\alpha i} R^*_{\beta i}. \tag{F8}$$

The parameter $\tilde{m}$ define in Eq. (36) can be straightforwardly identified as $\tilde{m} = \tilde{M}_{11}$ and, consequently,

$$\tilde{m} = \sum_i m_i |R_{1i}|^2 \geq \sum_i m_i R_{1i}^2 \geq m_1 \sum_i R_{1i}^2 = m_1. \tag{F9}$$

APPENDIX G: THE DAVIDSON-IBARRA BOUND [36]

We want to find the extrema of

$$f = \frac{\sum m_j^2 \mathcal{I}m R_{1j}^2}{\sum m_j |R_{1j}|^2}. \tag{G1}$$

Take $R_{1j}^2 = x_j + i y_j$. Then, orthogonality implies $x_1 + x_2 + x_3 = 1$, $y_1 + y_2 + y_3 = 0$:

$$f = \frac{(m_3^2 - m_1^2) y_3 + (m_2^2 - m_1^2) y_2}{m_3\sqrt{x_3^2 + y_3^2} + m_2\sqrt{x_2^2 + y_2^2} + m_1\sqrt{(1 - x_2 - x_3)^2 + (y_2 + y_3)^2}}. \tag{G2}$$

The largest $|f|$ will be reached when the denominator is smallest, which corresponds to $x_2 = 0$, $x_3 = 0$.

$$|f| \leq \left| \frac{(m_3^2 - m_1^2)y_3 + (m_2^2 - m_1^2)y_2}{m_3|y_3| + m_2|y_2| + m_1\sqrt{1 + (y_2 + y_3)^2}} \right|. \tag{G3}$$

The signs of y_2 and y_3 can be either the same or opposite. When they are the same,

$$\begin{aligned} |f| &\leq \frac{(m_3^2 - m_1^2)|y_3| + (m_2^2 - m_1^2)|y_2|}{m_3|y_3| + m_2|y_2| + m_1\sqrt{1 + (|y_2| + |y_3|)^2}} \\ &= \frac{(m_3^2 - m_1^2) + (m_2^2 - m_1^2)|z|}{m_3 + m_2|z| + m_1\sqrt{\frac{1}{|y_3|^2} + (|z| + 1)^2}}, \end{aligned} \tag{G4}$$

with $|z| = |y_2|/|y_3|$. Again, the maximum is reached when the denominator is minimal, which corresponds to $|y_3| \to \infty$. So,

$$|f| \leq \frac{(m_3^2 - m_1^2) + (m_2^2 - m_1^2)|z|}{m_3 + m_2|z| + m_1(|z| + 1)}. \tag{G5}$$

This function decreases monotonically in $0 \leq |z| < \infty$, so the extrema are in the boundaries. If we take $|z| = 0$, we obtain $|f| \leq m_3 - m_1$ and if we take $|z| \to \infty$, $|f| \leq m_2 - m_1$. The absolute maximum for this case is $|f| \leq m_3 - m_1$.

When the signs are opposite, from Eq.(G3) we obtain

$$\begin{aligned} |f| &\leq \left| \frac{(m_3^2 - m_1^2)|y_3| - (m_2^2 - m_1^2)|y_2|}{m_3|y_3| + m_2|y_2| + m_1\sqrt{1 + (|y_2| - |y_3|)^2}} \right| \\ &= \left| \frac{(m_3^2 - m_1^2) - (m_2^2 - m_1^2)|z|}{m_3 + m_2|z| + m_1\sqrt{\frac{1}{|y_3|^2} + (|z| - 1)^2}} \right| \\ &\leq \frac{|(m_3^2 - m_1^2) - (m_2^2 - m_1^2)|z||}{m_3 + m_2|z| + m_1||z| - 1|}, \end{aligned} \tag{G6}$$

where $|z| = |y_2/y_3|$ and where we have taken $|y_3| \to \infty$ to minimize the denominator (and maximize $|f|$).

Depending on the value of $|z|$, this function can take different values:

- $0 < |z| < 1$,

$$|f| \leq \frac{(m_3^2 - m_1^2) - (m_2^2 - m_1^2)|z|}{m_3 + m_2|z| + m_1(1 - |z|)}. \tag{G7}$$

- $1 < |z| < (m_3^2 - m_1^2)/(m_2^2 - m_1^2)$,

$$|f| \leq \frac{(m_3^2 - m_1^2) - (m_2^2 - m_1^2)|z|}{m_3 + m_2|z| + m_1(|z| - 1)}. \tag{G8}$$

- $(m_3^2 - m_1^2)/(m_2^2 - m_1^2) < z < \infty$,

$$|f| \le \frac{(m_2^2 - m_1^2)|z| - (m_3^2 - m_1^2)}{m_3 + m_2|z| + m_1(|z| - 1)}. \tag{G9}$$

In none of the intervals there is an extremum. Therefore, the extrema have to lie on the boundaries:

$$\begin{aligned} |z| = 0 &\implies |f| \le m_3 - m_1, \qquad \text{(G10)} \\ |z| = 1 &\implies |f| \le m_3 - m_2, \\ |z| = \frac{m_3^2 - m_1^2}{m_2^1 - m_1^2} &\implies |f| \le 0, \\ |z| = \infty &\implies |f| \le m_2 - m_1. \end{aligned}$$

The absolute maximum is again $|f| \le m_3 - m_1$.

Note that the bound is saturated when $|y_3| \to \infty$, which implies non-perturbative Yukawa couplings. Imposing perturbativity, the bound would become somewhat stronger. Also, imposing acceptable washout makes the bound stronger.

APPENDIX H: THE FAILURE OF THE DI BOUND IN THE $3 + 4$ CASE

This section is based on private communication with A. Strumia.

If there are more than three singlet fermions that have Yukawa couplings to the lepton doublets, then we cannot follow the derivation of the previous subsection since $RR^T \neq 1$. Indeed, the DI bound does not hold in this case, as demonstrated by the following example. The Lagrangian is given by

$$-\mathcal{L} = (g_1 N_1 + g_4 N_4) L_e H + g_2 N_2 L_\mu H + g_3 N_3 L_\tau H + \frac{1}{2} M_\alpha N_\alpha N_\alpha. \tag{H1}$$

The Yukawa matrix is then given as

$$\lambda = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \\ g_4 & 0 & 0 \end{pmatrix}. \tag{H2}$$

The see-saw relation gives us

$$m_\nu = v^2 \begin{pmatrix} \frac{g_1^2}{M_1} + \frac{g_4^2}{M_4} & 0 & 0 \\ 0 & \frac{g_2^2}{M_2} & 0 \\ 0 & 0 & \frac{g_3^2}{M_3} \end{pmatrix}. \tag{H3}$$

To get degenerate neutrinos, we need to have

$$\left|\frac{g_2^2}{M_2}\right| = \left|\frac{g_3^2}{M_3}\right| = \left|\frac{g_1^2}{M_1} + \frac{g_4^2}{M_4}\right| = \frac{m}{v^2}. \tag{H4}$$

The matrix U diagonalizes $m_\nu^\dagger m_\nu$ so in this case it is just the unit matrix. The diagonal D_ν matrix is given by

$$D_\nu = \begin{pmatrix} me^{i\alpha} & 0 & 0 \\ 0 & me^{i\beta} & 0 \\ 0 & 0 & me^{i\gamma} \end{pmatrix}. \tag{H5}$$

The matrix R is given by

$$R_{ij} = v \begin{pmatrix} \frac{g_1\sqrt{e^{-i\alpha}}}{\sqrt{M_1 m}} & 0 & 0 \\ 0 & \frac{g_2\sqrt{e^{-i\beta}}}{\sqrt{M_2 m}} & 0 \\ 0 & 0 & \frac{g_3\sqrt{e^{-i\gamma}}}{\sqrt{M_3 m}} \\ \frac{g_4\sqrt{e^{-i\alpha}}}{\sqrt{M_4 m}} & 0 & 0 \end{pmatrix}. \tag{H6}$$

We can indeed verify that $R^T R = 1$ since

$$R^T R = \frac{v^2}{m} \begin{pmatrix} e^{-i\alpha}(\frac{g_1^2}{M_1} + \frac{g_4^2}{M_4}) & 0 & 0 \\ 0 & e^{-i\beta}\frac{g_2^2}{M_2} & 0 \\ 0 & 0 & e^{-i\gamma}\frac{g_3^2}{M_3} \end{pmatrix} = 1. \tag{H7}$$

However,

$$(RR^T)_{11} = v^2 \frac{g_1^2 e^{-i\alpha}}{M_1 m}. \tag{H8}$$

We see that in order to have $\mathcal{I}m(RR^T)_{11} = 0$, then the phase of g_1^2 is α. However, the phase of $g_1^2 + g_4^2$ must also be α from eq. (H7). This can only be if g_4^2 has a phase α. But this necessarily means that the CP asymmetry will vanish since the CP asymmetry $\epsilon \propto \mathcal{I}m((g_1 g_4^*)^2)$. Therefore, in this example, we cannot have $\mathcal{I}m(RR^T)_{11} = 0$ and the proof of the DI bound fails.

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